Homogenizacja numeryczna

Włókno szklane:
E_w = 73500 MPa
v_w = 0,22

Spoiwo epoksydowe:
E_s = 3400 MPa
v_s = 0,35
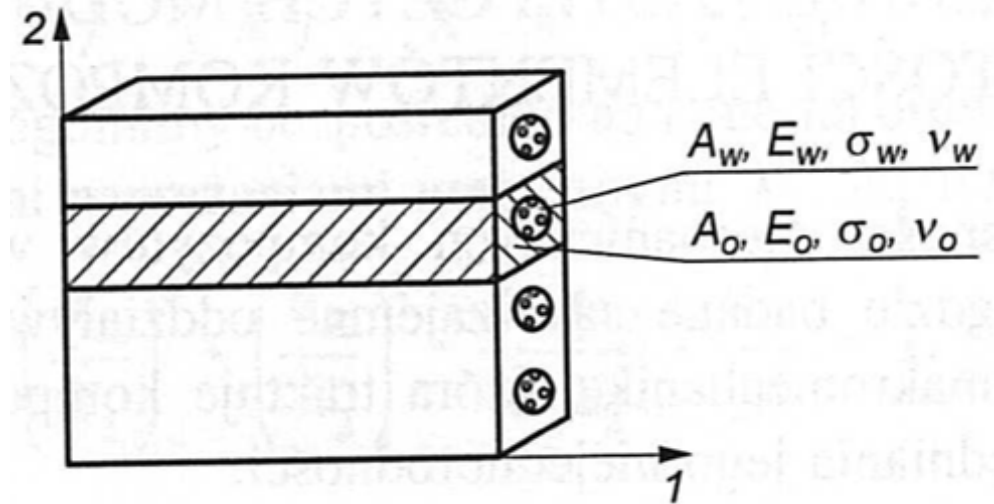
$V_w = 0,44^2\pi/1 \approx 0,61$
$V_s \approx 0,39$

$$G = \frac{E}{2(1+\nu)} \rightarrow G_w = \frac{73500}{2 \cdot 1{,}22} = 30123 \text{ MPa}; \quad G_s = \frac{3400}{2 \cdot 1{,}35} = 1259 \text{ MPa}$$
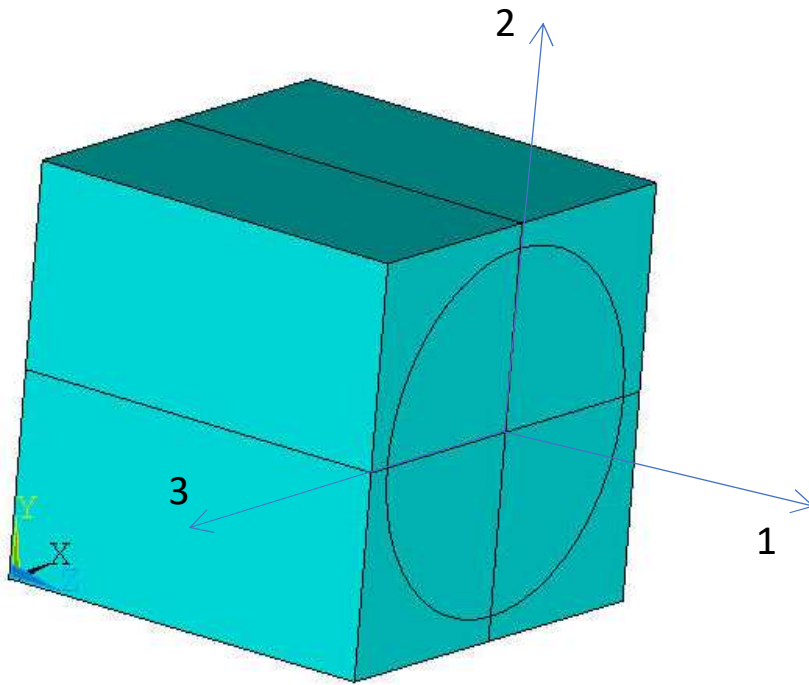
$$E_1 = E_w V_w + E_s V_s = 46161 \text{ MPa}$$

$$E_2 = \frac{E_w \cdot E_s}{E_s V_w + E_w V_s} = 8130 \text{ MPa}$$

$$\nu_{12} = \nu_w V_w + \nu_s V_s = 0{,}27$$

$$G_{12} = \frac{G_w \cdot G_s}{G_s V_w + G_w V_s} = 2010 \text{ MPa}$$

$A_w, E_w, \sigma_w, \nu_w$

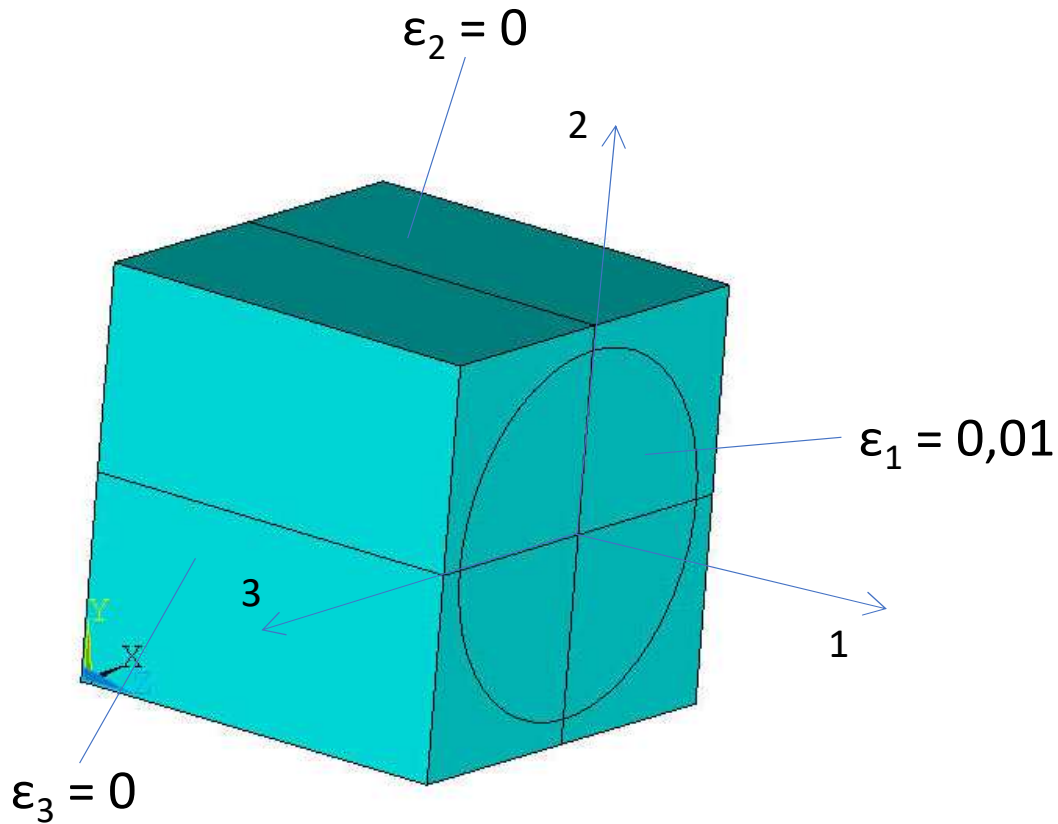$A_o, E_o, \sigma_o, \nu_o$

$E_2 = E_3$

$v_{12} = v_{13}$

$G_{12} = G_{13}$

Zależność między odkształceniem i naprężeniem

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{\nu_{12}}{E_1} & -\dfrac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\dfrac{\nu_{12}}{E_1} & \dfrac{1}{E_2} & -\dfrac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\dfrac{\nu_{13}}{E_1} & -\dfrac{\nu_{23}}{E_2} & \dfrac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

$$
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{\nu_{12}}{E_1} & -\dfrac{\nu_{13}}{E_1} \\[2mm] -\dfrac{\nu_{12}}{E_1} & \dfrac{1}{E_2} & -\dfrac{\nu_{23}}{E_2} \\[2mm] -\dfrac{\nu_{13}}{E_1} & -\dfrac{\nu_{23}}{E_2} & \dfrac{1}{E_3} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}
\quad \Longrightarrow \quad
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{\nu_{12}}{E_1} & -\dfrac{\nu_{12}}{E_1} \\[2mm] -\dfrac{\nu_{12}}{E_1} & \dfrac{1}{E_2} & -\dfrac{\nu_{23}}{E_2} \\[2mm] -\dfrac{\nu_{12}}{E_1} & -\dfrac{\nu_{23}}{E_2} & \dfrac{1}{E_2} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}
$$

$$
\Downarrow
$$

$$
E_1,\ E_2,\ E_3,\ \nu_{12},\ \nu_{13},\ \nu_{23}
$$

$\varepsilon_2 = 0$

2

$\varepsilon_1 = 0{,}01$
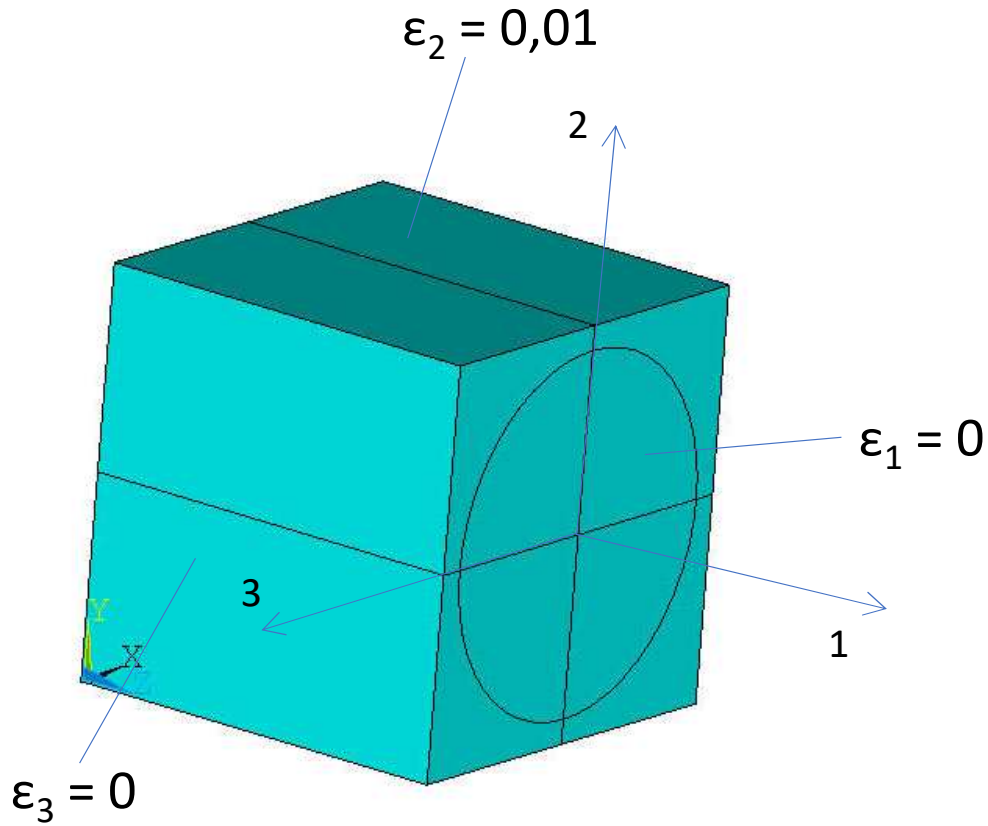
3

1

$\varepsilon_3 = 0$

$\sigma_1', \sigma_2', \sigma_3'$

$$\begin{Bmatrix} 0{,}01 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{\nu_{12}}{E_1} & -\dfrac{\nu_{13}}{E_1} \\ -\dfrac{\nu_{12}}{E_1} & \dfrac{1}{E_2} & -\dfrac{\nu_{23}}{E_2} \\ -\dfrac{\nu_{13}}{E_1} & -\dfrac{\nu_{23}}{E_2} & \dfrac{1}{E_3} \end{bmatrix} \begin{Bmatrix} \sigma_1' \\ \sigma_2' \\ \sigma_3' \end{Bmatrix}$$

$$0{,}01 = \frac{1}{E_1}\sigma_1' - \frac{\nu_{12}}{E_1}\sigma_2' + \frac{\nu_{13}}{E_1}\sigma_3'$$

$$0 = -\frac{\nu_{12}}{E_1}\sigma_1' + \frac{1}{E_2}\sigma_2' - \frac{\nu_{23}}{E_2}\sigma_3'$$

$$0 = -\frac{\nu_{13}}{E_1}\sigma_1' - \frac{\nu_{23}}{E_2}\sigma_2' + \frac{1}{E_3}\sigma_3'$$

$$\begin{Bmatrix} 0 \\ 0,01 \\ 0 \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{\nu_{12}}{E_1} & -\dfrac{\nu_{13}}{E_1} \\[3mm] -\dfrac{\nu_{12}}{E_1} & \dfrac{1}{E_2} & -\dfrac{\nu_{23}}{E_2} \\[3mm] -\dfrac{\nu_{13}}{E_1} & -\dfrac{\nu_{23}}{E_2} & \dfrac{1}{E_3} \end{bmatrix} \begin{Bmatrix} \sigma_1'' \\ \sigma_2'' \\ \sigma_3'' \end{Bmatrix}$$

$$0 = \frac{1}{E_1}\sigma_1'' - \frac{\nu_{12}}{E_1}\sigma_2'' + \frac{\nu_{13}}{E_1}\sigma_3''$$

$$0,01 = -\frac{\nu_{12}}{E_1}\sigma_1'' + \frac{1}{E_2}\sigma_2'' - \frac{\nu_{23}}{E_2}\sigma_3''$$

$$0 = -\frac{\nu_{13}}{E_1}\sigma_1'' - \frac{\nu_{23}}{E_2}\sigma_2'' + \frac{1}{E_3}\sigma_3''$$
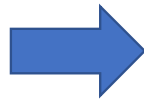
$$\sigma'_1 = 449{,}19 \ MPa$$

$$\sigma'_2 = 59{,}37 \ MPa$$

$$\sigma'_3 = 59{,}34 \ MPa$$

$$\sigma''_1 = 61{,}62 \ MPa$$

$$\sigma''_2 = 176{,}69 \ MPa$$

$$\sigma''_3 = 50{,}82 \ MPa$$

$$0{,}01 = \frac{1}{E_1}\sigma'_1 - \frac{\nu_{12}}{E_1}\sigma_2{'} + \frac{\nu_{13}}{E_1}\sigma_3{'}$$

$$0 = -\frac{\nu_{12}}{E_1}\sigma'_1 + \frac{1}{E_2}\sigma'_2 - \frac{\nu_{23}}{E_2}\sigma'_3$$

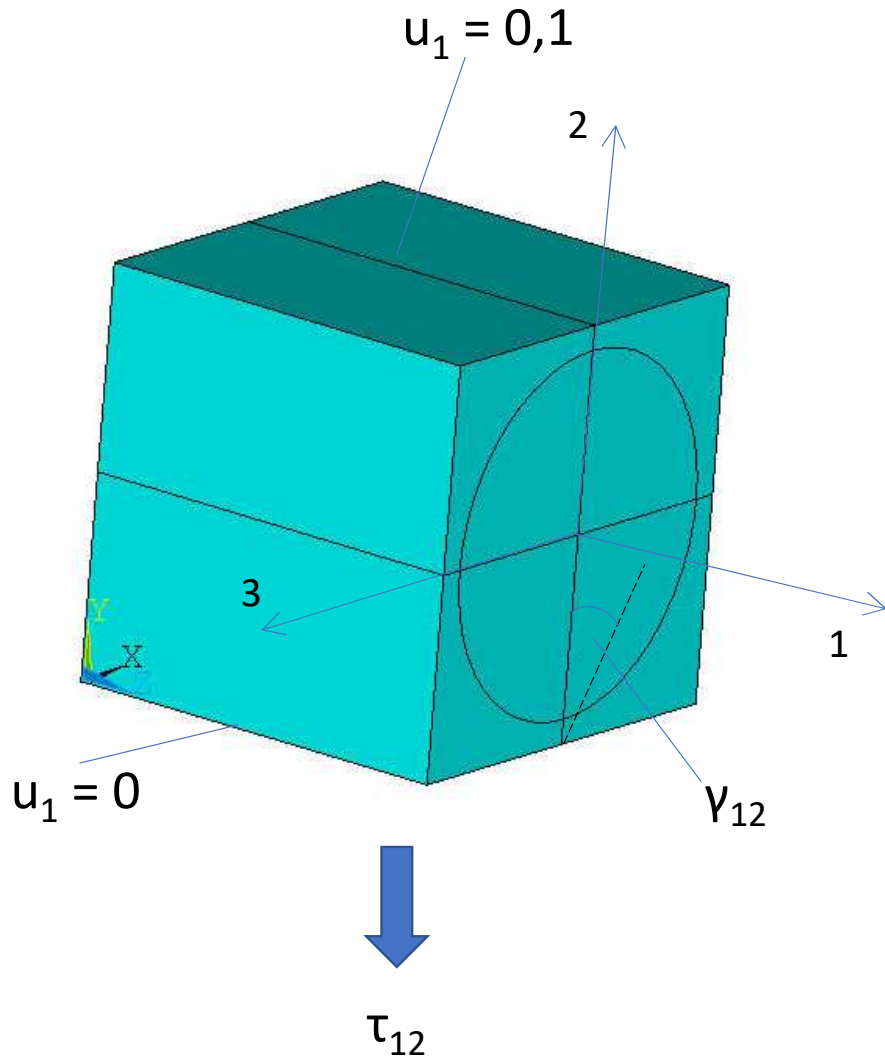$$0 = -\frac{\nu_{13}}{E_1}\sigma'_1 - \frac{\nu_{23}}{E_2}\sigma'_2 + \frac{1}{E_3}\sigma'_3$$

$$0 = \frac{1}{E_1}\sigma''_1 - \frac{\nu_{12}}{E_1}\sigma_2{''} + \frac{\nu_{13}}{E_1}\sigma_3{''}$$

$$0{,}01 = -\frac{\nu_{12}}{E_1}\sigma''_1 + \frac{1}{E_2}\sigma''_2 - \frac{\nu_{23}}{E_2}\sigma''_3$$

$$0 = -\frac{\nu_{13}}{E_1}\sigma''_1 - \frac{\nu_{23}}{E_2}\sigma''_2 + \frac{1}{E_3}\sigma''_3$$

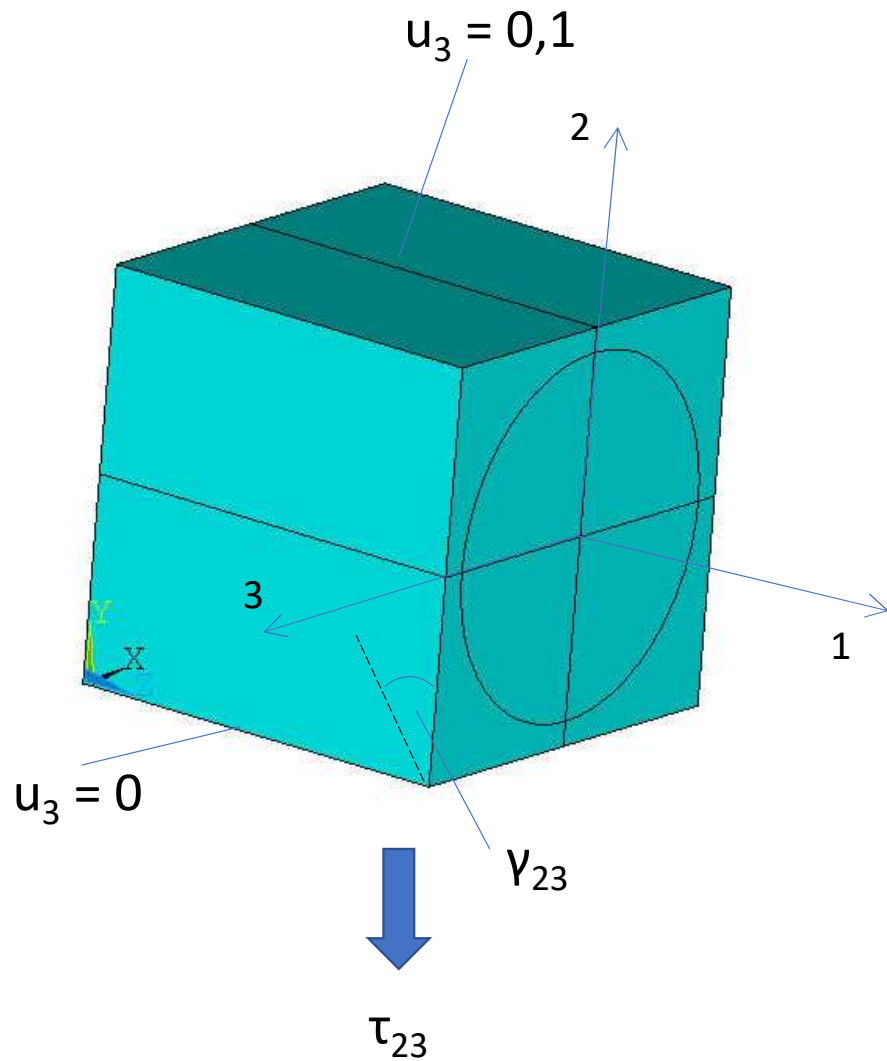|	Homogenizacja analityczna	Homogenizacja numeryczna
E_1	46161 MPa	36307 MPa
E_2	8130 MPa	15294 MPa
E_3	8130 MPa	14810 MPa
ν_{12}	0,27	0,26
ν_{23}	-	0,14
ν_{13}	0,27	0,25

$$tg\gamma_{12} = \frac{0,1}{1} \rightarrow \gamma_{12} = arctg\,0,1$$

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$
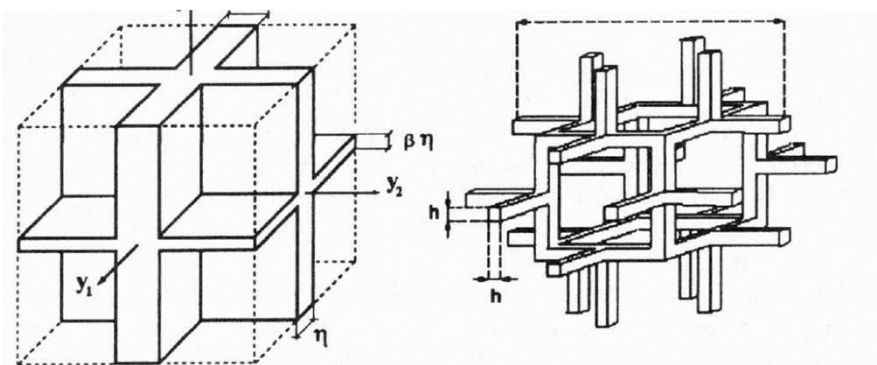
$$\tau_{12} = \tau_{13} = 124,19 \; MPa$$

Labels in figure:

$u_1 = 0,1$

2

3

1

X

$u_1 = 0$

γ_{12}

τ_{12}

$$tg\gamma_{23} = \frac{0,1}{1} \rightarrow \gamma_{23} = arctg\,0,1$$

$$G_{23} = \frac{\tau_{23}}{\gamma_{23}}$$

$$\tau_{23} = 134,82\ MPa$$

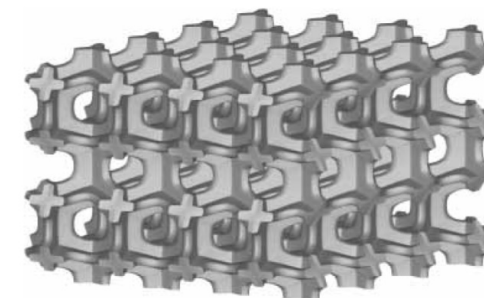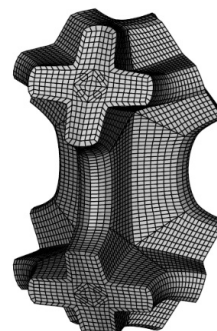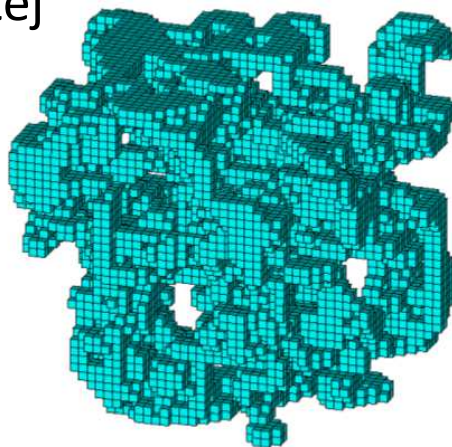|	Homogenizacja analityczna	Homogenizacja numeryczna
G_{12}	2010 MPa	1246 MPa
G_{13}	2010 MPa	1246 MPa
G_{23}	–	1352 MPa

Przykłady zaawansowanej homogenizacji numerycznej

model do analitycznej homogenizacji
periodycznej struktury porowatej

model numeryczny
struktury periodycznej

model numeryczny struktury chaotycznej