Integrated Laboratory

Strength of Materials and Structures

Buckling

Before attending the laboratory students should recollect the following topics: buckling of a rod, critical force, slenderness ratio, radius of gyration, axial and eccentric load, Euler's formula

Recommended Bibliography:

- William A. Nash *Strenght of materials*
- Roy R. Craig *Mechanics of Materials*
- *Mechanika Materiałów i Konstrukcji* edited by Marek Bijak-Żochowski
- Own lecture notes
- the Internet (for lazy students)

1 The Objectives of the Works

This exercise is divided into two sections: obligatory and supplementary.

In the obligatory part students have to prove two thesis:

1. *thesis 1*: critical force does not depend on shear force
2. *thesis 2*: critical force does depend on operation of loading forces during the process of buckling

In the supplementary part students have to study:

3. buckling of the spring
4. lateral-torsional buckling of the beam
5. buckling of the truss

2 Basic Knowledge

2.1 Buckling of the Ideal Rod

It is known, that in some conditions, straight rod could have two forms of balance: original form (simply straight rod) and the secondary one (curved one). This phenomenon can be observed when compressive load (ideally axially compression) reaches certain value - called critical value.

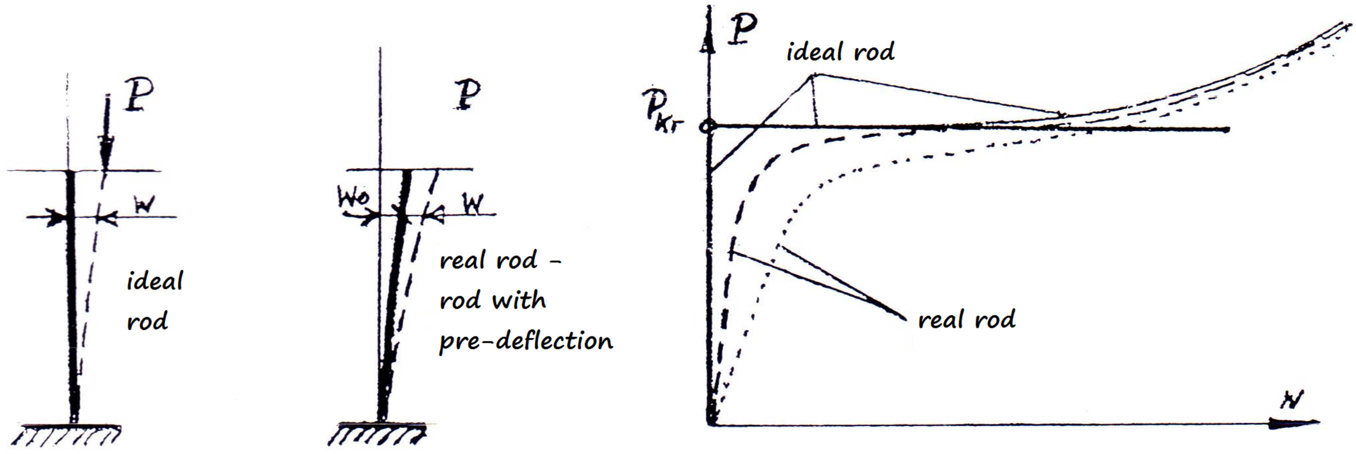
The critical load is a certain value - it determines the state of a rod:

value of compression load	**state of equilibrium**
value of compression load **less** than critical value	stable equilibrium
value of compression load **equals** critical value	neutral equilibrium
value of compression load **more** than critical value	unstable equilibrium

It is important to understand the neutral equilibrium. Imagine that rod undergoes the critical compressive load. Than we add another load. If we remove this second load, the rod neither return to his first form nor deepen the deflection.

In the picture 1 (a) we can see ideally straight rod, ideally axially compressed (ideal situation). In fact, the axis of rod is not ideally straight and the compression is not ideally axial. This causes shear force Q and bending moment (result of eccentric operation of the loading force). In the picture 1 (b) we can see this problem.

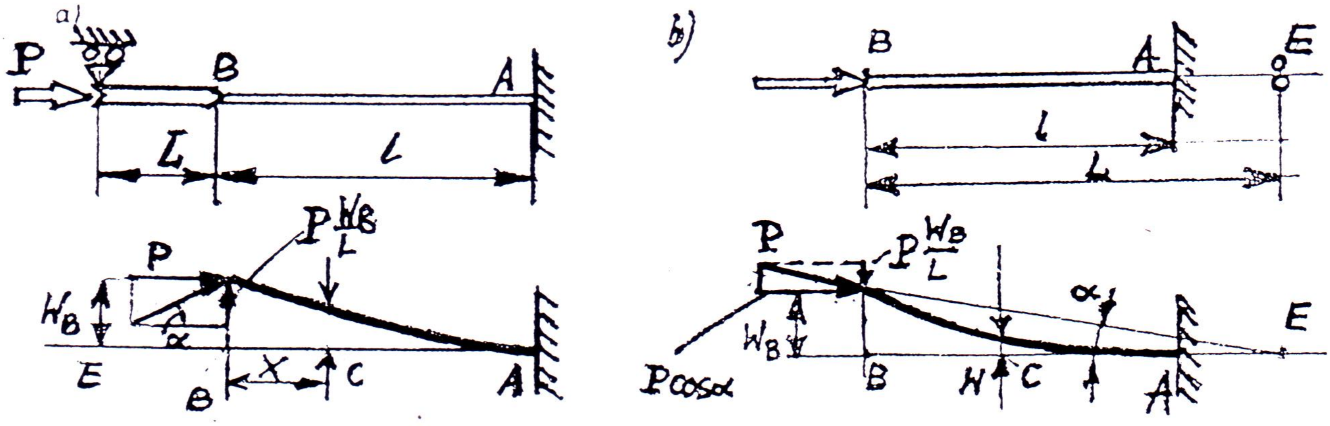
Real rods operate different than ideal ones. The axis of the rod became curved once the compressive load occurs. The nature of the deflection can be observed in the picture 1 (c). In the picture ordinate corresponding to the critical load is an asymptote.



Picture 1: *Compressed rods: (a) ideal rod, (b) rod with pre-deflection, (c) real rod*

We can postulate a theorem:
effect of lateral loads do not change the critical value $P_{critical}$ of the main load P. This fact is proven both theoretically and experimentally.

In some cases it is difficult to test the stability of an element because there is a change in direction of loading force during operation. Such situation can be seen in pictures 2 (a) and (b). Because of the buckling the direction of the force varies by α.



Picture 2: *Compressed rods; change in direction of loading force during operation*

We can postulate a theorem:
The value of critical load does depend on the nature of loading force during buckling.

2.2 How to Estimate the Critical Load of the Ideal Rod Experimentally

Let's assume there is a shear force Q.

Deflection f of shear force Q and also main forces P is:

$$\frac{f_q}{1 - \frac{P}{P_{critical}}},$$

where f_Q is deflection caused by shear force Q.

If shear force Q is proportional to main load P:

$$Q = K * P$$

then

$$f_Q = b_Q * P$$

so we have:

$$f = \frac{b_Q * P_{critical}}{\frac{P_{critical}}{P} - 1} = \frac{C_1}{\frac{P_{critical}}{P} - 1}$$

where constant b_Q has to have appropriate value, and $C_1 = b_Q * P_{critical} = const$

There is an initial deflection f_0. So we have an extra deflection f caused by main compressive load P:

$$f = \frac{f_0}{\frac{P_{critical}}{P} - 1}$$

Above formulas are used to determine $P_{critical}$ experimentally. Experiment should follow below steps:

- determine the deflection f for two values of P
- substitute two values of P and f to the appropriate equation

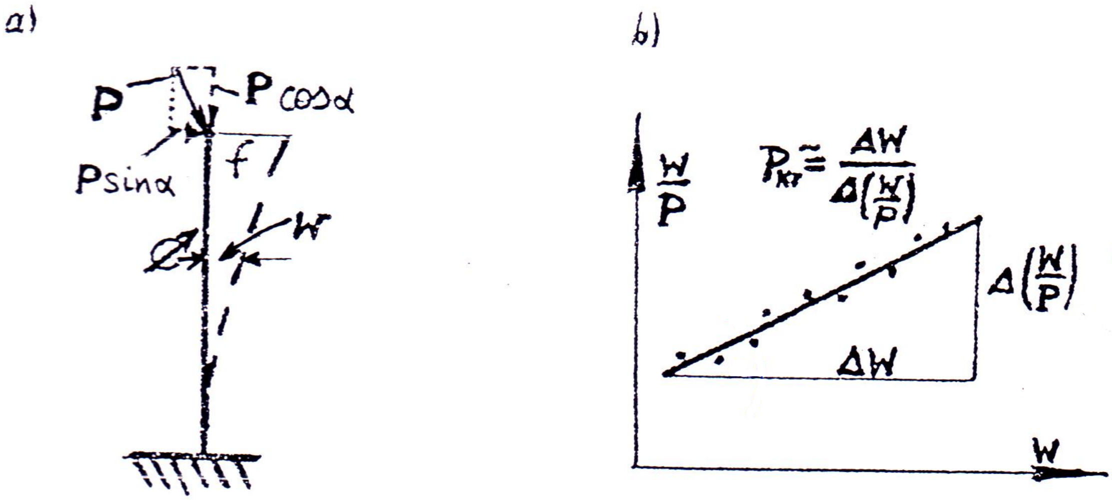
Now we have system of equations with two unknowns: $P_{critical}$ and C. We can solve this system and get $P_{critical}$.

To increase the accuracy of the result we should determine deflection f more than two times. We make plots $\frac{f}{P}$ and f. Our results should create a straight line, because:

$$P_{critical}(\frac{f}{P}) - f = C_2 = const$$

If we want to estimate the critical value of compressive load, we would find the slope of the line. Let's assume that deflection is proportional. Then:

$$P_{critical}(\frac{w}{P}) - w = C_2 = const$$



Picture 3: *(a) Scheme of compressed rod, (b) plot*

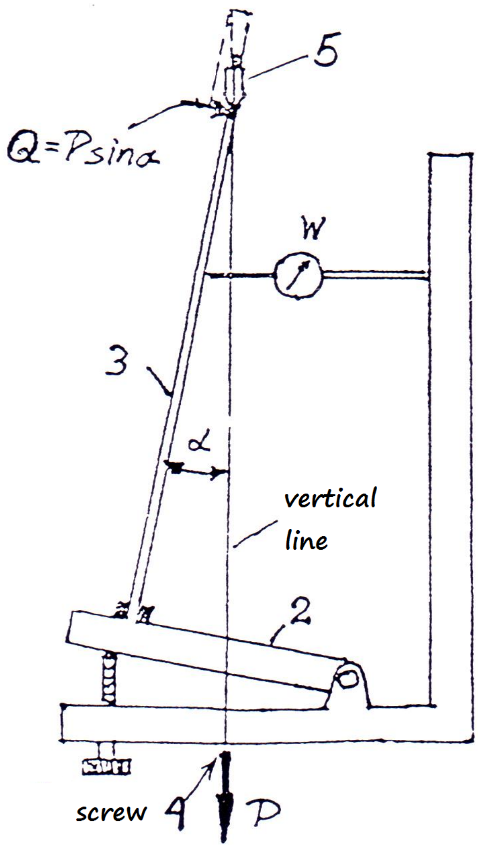
After giving points from measurement on the plot (shown in the picture 3 (b)) we find the critical value as:

$$P_{critical} = \frac{\Delta w}{\Delta(\frac{P}{w})}$$

3 About the test bench

The scheme of test bench is shown in the picture 4. There is steel rod (3) on the frame (2) of construction. This rod can be loaded by weight (4) directly, or by connector (5). If we load the rod by weight, we are dealing with the situation shown in the picture 1 (a). And in the second case, the situation is shown in the picture 2 (a). Moreover, when we both load the rod by weight and set the ties of weight in one point, we have the situation shown in the picture 2 (b).

Furthermore, there are spring and beam on the trunk. Deflections of this two elements could be measured by dial indicator. In this two cases the load is added in the same way: weights (typically: $5kG$) are put on weighting pan (10).



Picture 4: *Test bench*

4 What to do

4.1 Proof of the *thesis 1*

We have to test the rod as it is shown in the picture 4 to prove the *thesis 1*. To create slight but firm shear force Q we have to turn aside rod (3) by rotating the frame (2) by screw. Every time the load P is changed, read the value from the indicator.

Use measured values to find the deflection w. Then find the critical value $P_{critical}$.

You should make three measurements: for three different angles. Measurement accuracy: $0.1mm$ for cross-section of the rod and $1mm$ for the length of the rod.

Compare these three calculated values of the critical force with each other and with the value calculated from Euler's formula.

Draw a conclusion about the thesis.

4.2 Proof of the *thesis 2*

We have to test the rod as it is shown in the picture 2 (a) and (b) to prove the *thesis 2*.

Measure the deflection w for some values of loading force P. Find the values of critical forces. Compare this values with the theoretical value.

You can find the theoretical formula in: *Z. Brzoska: Wytrzymałość materiałów, p.15.1*

5 Report

Report should consist of:

1. Brief view of the topic and short description of the test bench
2. Proof of the *thesis 1*: how the test was performed and all the measurements from the test
   (a) table with measurements
   (b) plots: $(\frac{w}{P}) = f(w)$
   (c) calculations of critical force $P_{critical}$ from the test
   (d) calculations of the theoretical value of critical force $P_{critical}$ from the test and required comparison
3. Proof of the *thesis 2*: how the test was performed and all the measurements from the test
   (a) table with measurements
   (b) plots: $(\frac{w}{P}) = f(w)$
   (c) calculations of critical force $P_{critical}$ from the test
   (d) calculations of theoretical value of critical force $P_{critical}$ from the test and required comparison
4. Proof of one of the concepts from supplementary part: how the test was performed and all the measurements from the test
   (a) table with measurements
   (b) plots: $(\frac{w}{P}) = f(w)$
   (c) calculations of critical force $P_{critical}$ from the test
   (d) calculations of theoretical value of critical force $P_{critical}$ from the test and required comparison
5. Summary with conclusions