LAB 2. Finite element analysis of free vibrations (modal analysis).

CASE 1.

<u>A cantilever beam with a C-section</u>

1. Geometry of the beam (Fig. 1).

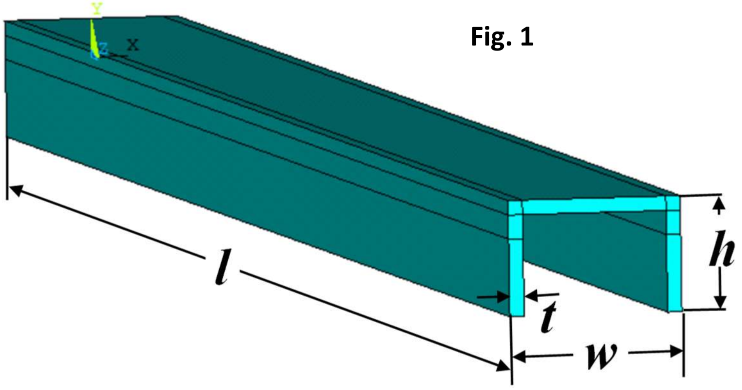


Fig. 1

a) create a text file and save it as *keypoints.txt* in your working directory:

```
! ______________MACRO TO DEFINE KEYPOINTS IN C-SECTION________________
!*
w=60                                     !   width along x-axis   (mm)
h=40                                     !   height along x-axis  (mm)
t=5                                      !   wall thickness         (mm)
y=(h*h+(w-2*t)*(h-t/2))/(2*(h-t)+w)      !   y coordinate of the center of gravity   (mm)
/PREP7                                   !  Main Menu/ Preprocessor
                    ! Main Menu/ Preprocessor/Modeling/Create/Keypoints/In Active CS
k,1,-w/2,-y,0,                           ! 1- no. of keypoint, X=-w/2, Y=-y, Z=0
k,2,-w/2+t,-y,0,
k,3,-w/2+t,0,0,
k,4,-w/2,0,0,
k,5,-w/2+t,h-y-t,0,
k,6,-w/2,h-y-t,0,
k,7,-w/2+t,h-y,0,
k,8,-w/2,h-y,0,
k,9,w/2,-y,0,
k,10,w/2-t,-y,0,
k,11,w/2-t,0,0,
k,12,w/2,0,0,
k,13,w/2-t,h-y-t,0,
k,14,w/2,h-y-t,0,
k,15,w/2-t,h-y,0,
k,16,w/2,h-y,0,
!* __________________________END OF MACRO ______________________________
```

b) Execute the macro (*Utility Menu>File>Read Input from… ->* *keypoints.txt)*

c) create 7 areas through the keypoints (Fig. 2): 1->2->3->4->OK, (3->5->6->4, 5->7->8->6, 5->13->15->7, 13->14->16->15, 11->12->14->13, 10->9->12->11

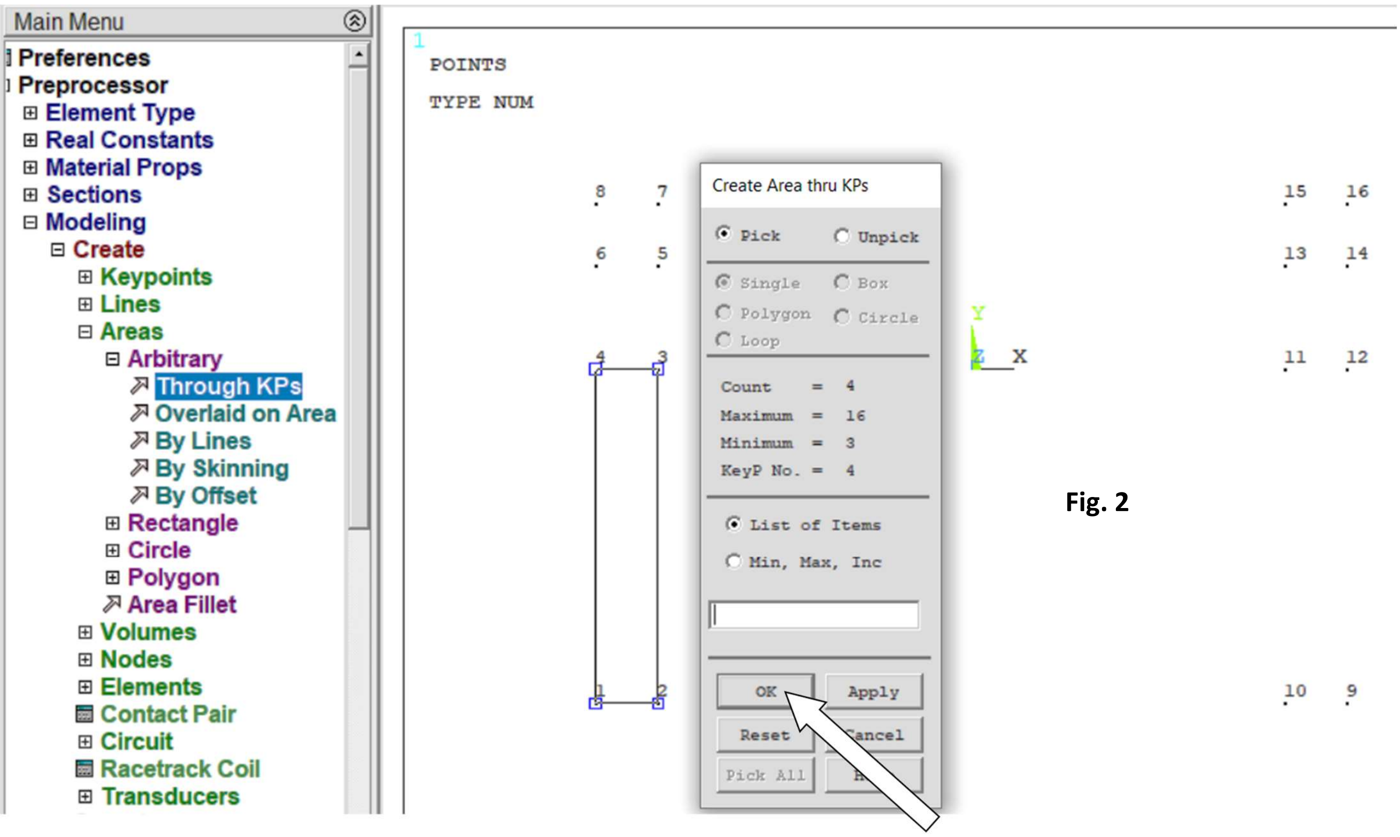


Fig. 2

d) check geometric properties of the cross-sectional area (*Utility Menu>List>Picked Entities>Area>Areas> PickAll*) (Fig. 3)

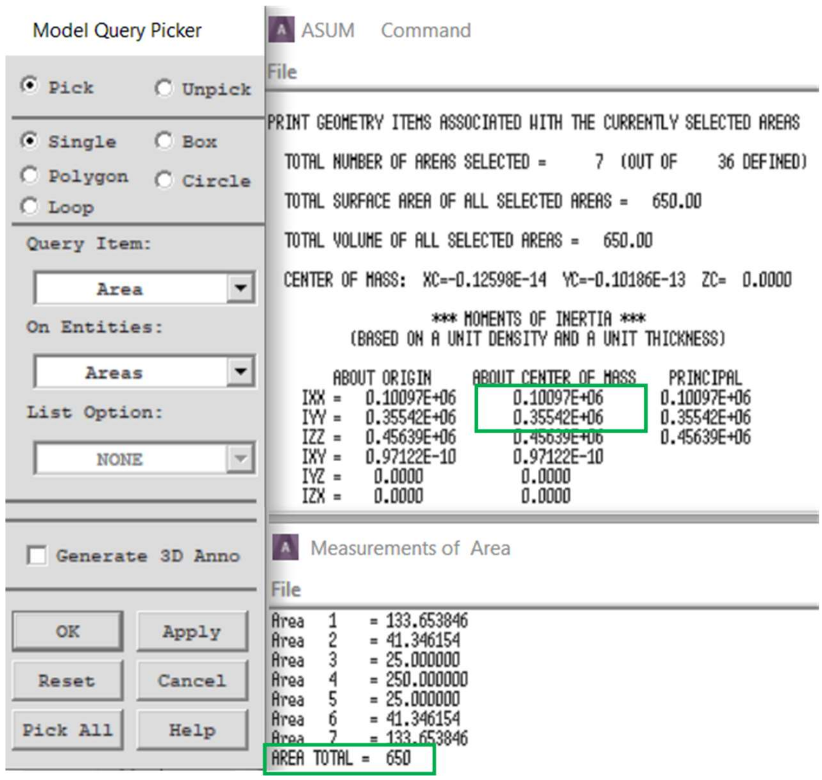


Fig. 3

e) extrude the cross-section along z-axis by distance l (*Preprocessor>Modeling>Operate>Areas>By XYZ Offset> Pick All*)

2. Choose SOLID185 finite element: (brick 8-node, element technology: Enhanced strain) (*Preprocessor>Element Type>Add/Edit/Delete*) (Fig. 4)

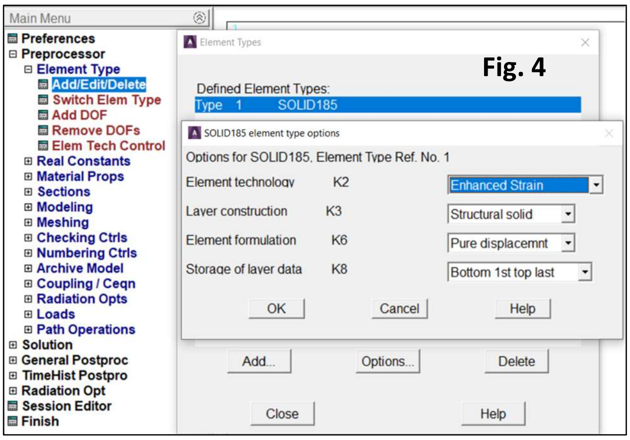


Fig. 4

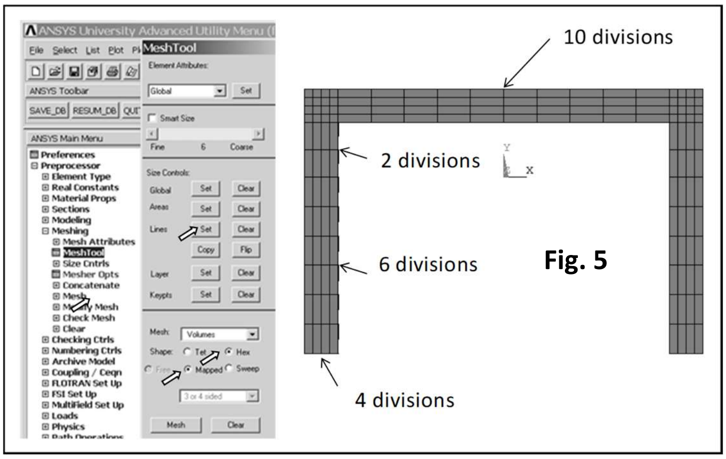


Fig. 5

3. Define material properties of a steel (linear elastic and isotropic) Young's modulus E = 2·10^5*MPa*, Poisson's ratio $\nu = 0.3$, density $\rho = 7.8 \cdot 10^{-9}$ Ns^2/mm^4 : (*Preprocessor>Material Props>Material Models> Structural)*

4. Specify the density of discretization on lines (100 divisions along z-axis) and create a mapped hexahedral mesh (*Preprocessor>Meshing> MeshTool)* (Fig. 5)

5. Save the mesh as an image (*Plot> Elements, PlotCtrls> Redirect Plots-> To JPEG File …*)

6. Save the database (*Utility Menu>File>Save As…*, for example: C_section_mesh.db)

7. Apply boundary conditions: Fixed support for the cross-section z = 0 (*Solution> Apply> Structural> Displacement> On Areas> All DOF*)

8. Choose modal analysis and define options (Fig. 6).

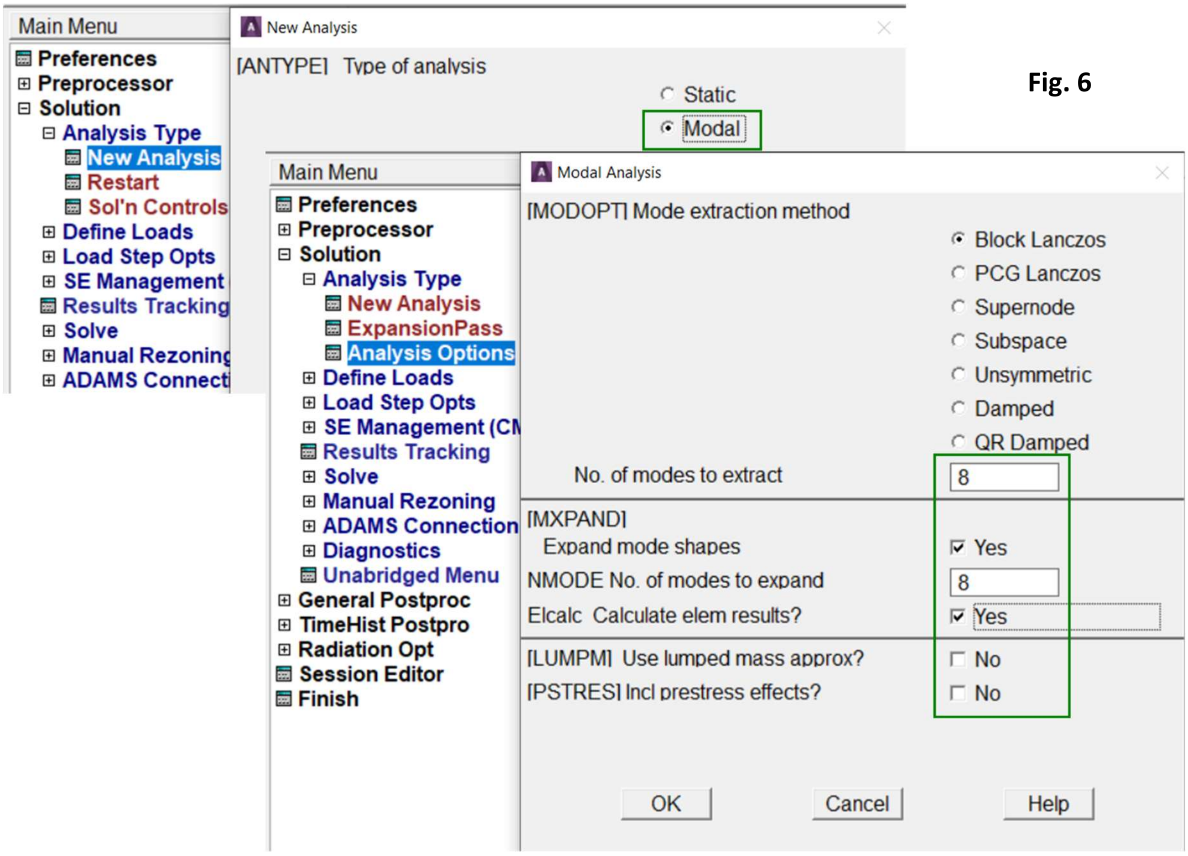


Fig. 6

9. Solve (*Solution-> Solve-> Current LS*)

10. Read results of modal analysis (*Main Menu> General Postproc> Read Results> By Pick*) for the first set (Fig. 7)

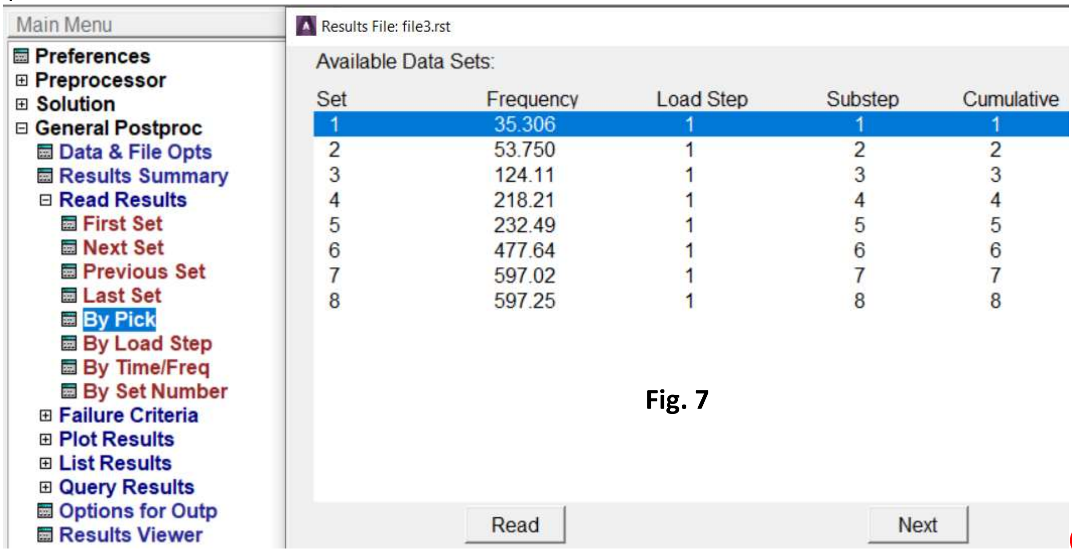


Fig. 7

(l = 1000 mm)

11. Plot animation of displacement in y direction (Utility Menu>*PlotCtrls>Animate>Deformed Results >DOF Solution>UY*) (Fig. 8).

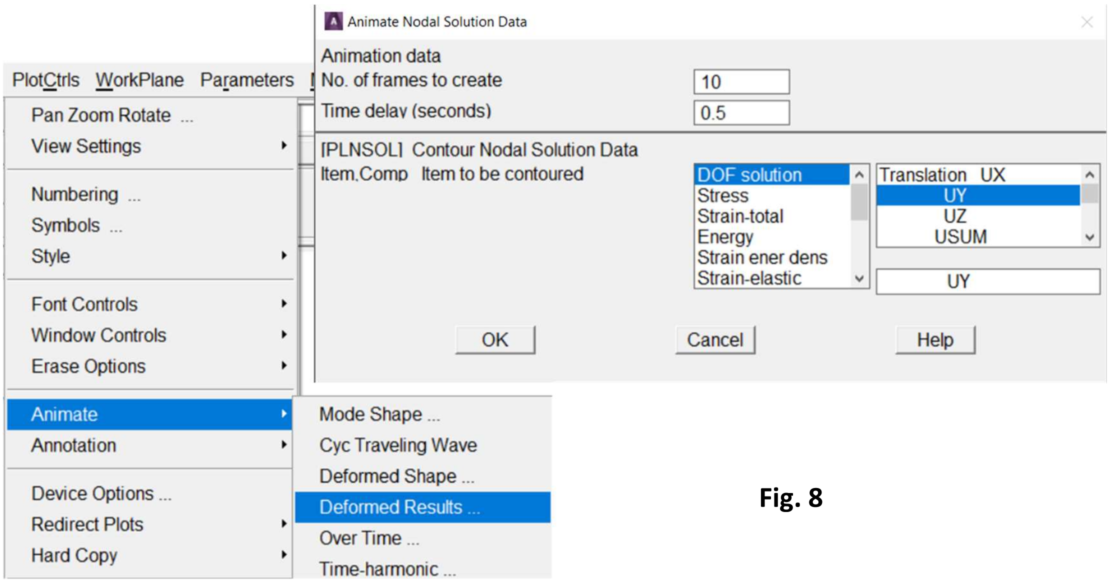
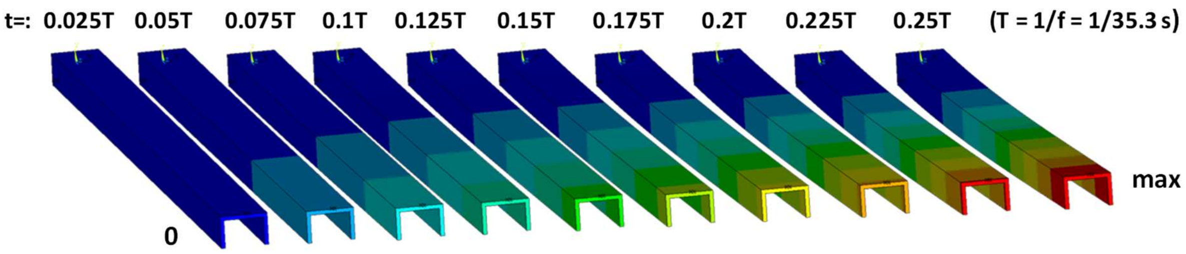


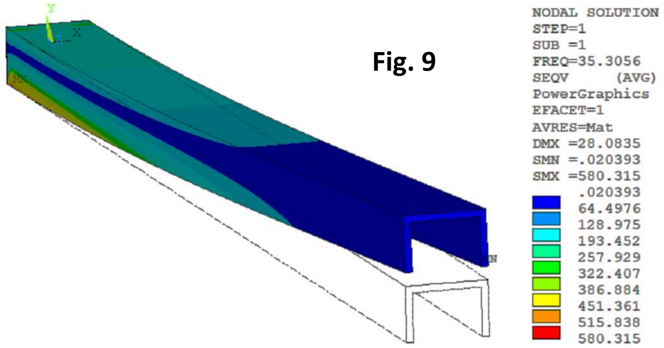
Fig. 8



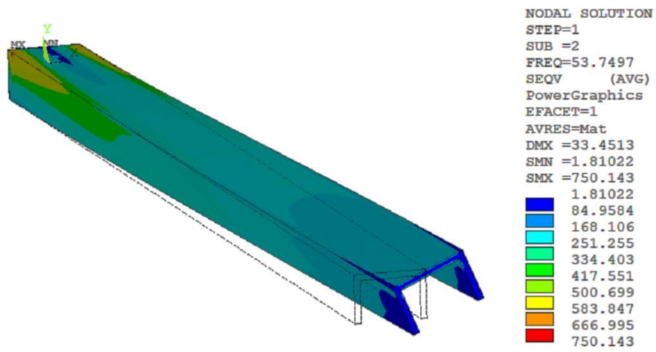
12. Plot contour maps of von Mises stress for all frequencies (magnitudes are not realistic), select the option: *Deformed shape with undeformed edge* (*Main Menu>General Postproc>Plot Results>Contour Plot>Nodal Solu> Stress>Von Mises*). Save each contour map as an image (Fig. 9) and fill Table. 1.

(Results for l = 1000 mm)

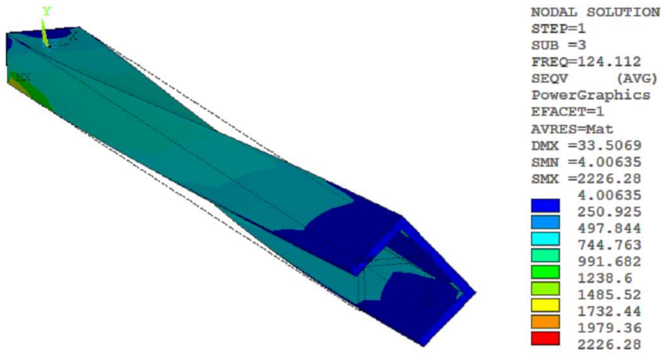
Fig. 9



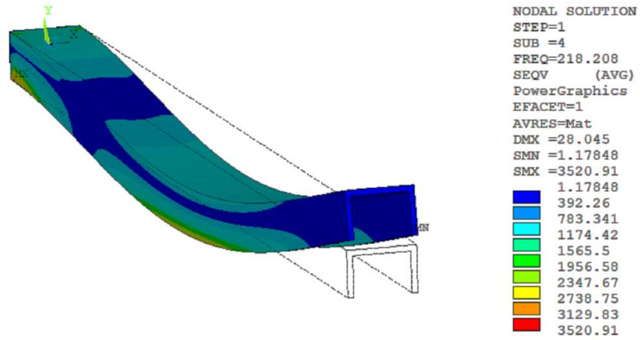
1st mode = 1st bending mode in YZ plane at 35.3 Hz



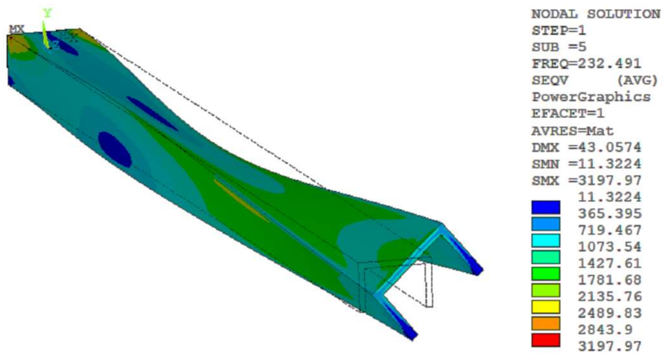
2nd mode = 1st bending- torsional mode at 53.7 Hz



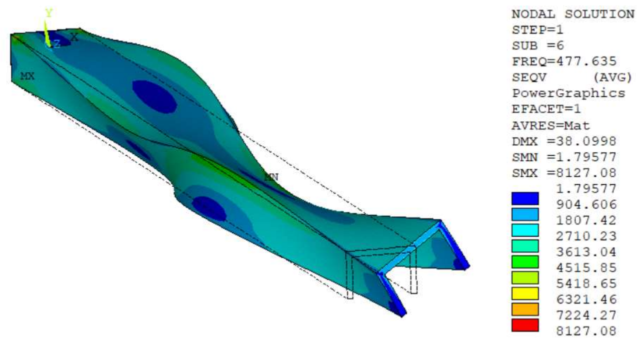
3rd mode = 2nd bending-torsional mode at 124.1 Hz



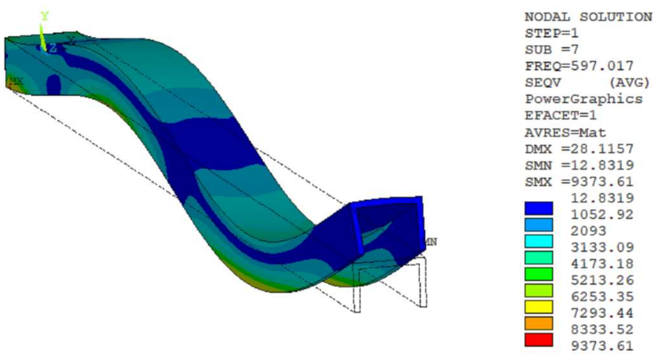
4th mode = 2nd bending mode in YZ plane at 218.2 Hz



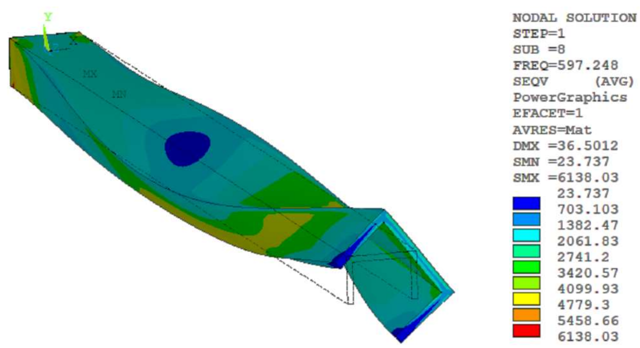
5th mode = 1st torsional mode at 232.5 Hz



6th mode = 3nd bending-torsional mode at 477.6 Hz



7th mode = 3rd bending mode in YZ plane at 579 Hz



8th mode = 4th bending-torsional mode at 579.2 Hz

Analytical solution: $c = \frac{1}{l^2}\sqrt{\frac{EJ}{\rho A}}$; $J = \begin{Bmatrix} IXX - \text{for bending in YZ plane} \\ IYY - \text{for bending in XZ plane} \end{Bmatrix}$

Type of support		f_1/c	f_2/c	f_i/c
$z = 0$	$z = l$			
fixed	fixed	3.560	9.815	$0.393(2i+1)^2$
fixed	pinned	2.454	7.951	$0.098(4i+1)^2$
fixed	free	0.560	3.506	$0.393(2i-1)^2$
pinned	pinned	1.571	6.283	$1.571i^2$

Example. Cantilever beam with length $l = 1000mm$:

1ST bending mode in YZ plane:

$J = IXX = 100970mm^4,\ A = 650mm^2, E = 200000\frac{N}{mm^2}, \rho = 7.810^{-9}Ns^2/mm^4$

$c = \frac{1}{1000^2}\sqrt{\frac{200000\cdot 100970}{7.810^{-9}\cdot 650}}$ $=63.11\frac{1}{s}$; $f_1 = \frac{f_1}{c}\cdot c = 0.56\cdot 63.11\frac{1}{s} = 35.34\text{Hz}$

relative error: $\Delta f_1 = \left|\frac{35.3056-35.34}{35.34}\right| = 0.1\%$

2nd bending mode in YZ plane: $f_2 = \frac{f_2}{c}\cdot c = 3.506\cdot 63.11\frac{1}{s} = 221.27\text{Hz}$

relative error: $\Delta f_2 = \left|\frac{218.2-221.27}{221.27}\right| = 1.4\%$

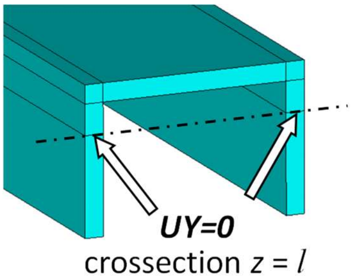
3rd bending mode in YZ plane: $f_3 = \frac{f_3}{c}\cdot c = 0.393\cdot(2\cdot 3-1)^2\cdot 63.11\frac{1}{s} = 620.07\text{Hz}$

relative error: $\Delta f_3 = \left|\frac{579.02-62\ \ .07}{620.07}\right| = 6.7\%$

CASE 2.

Beam with fixed cross-section at z = 0 and pinned cross-section at z = l

Fig. 10

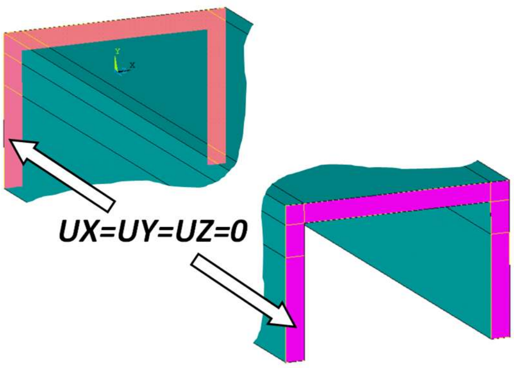


Plot contour maps of von Mises stress for 8 frequencies (see point 12). Save contour maps as images and fill Table. 2.

CASE 3.

Beam with fixed cross-sections

Fig. 11



Plot contour maps of von Mises stress for 8 frequencies (see point 12). Save contour maps as images and fill Table. 3.

CASE 4.

Rotating cantilever beam (with fixed cross-section at z = 0 and free cross-section at z = l)

The stress state in a structure changes its natural frequencies. To include this effect in a modal analysis we perform a static analysis with the prestress option.

13. Start a new analysis (static).
14. Change *Unabrigded Menu* to *Abridged Menu* (*Main Menu>Solution*)
15. Enable the prestress effect (*Solution>Analysis Type>Analysis Options>Prestress>On)* (Fig. 12).
16. Apply constraints (fixed cross-section at z = 0, the same as for CASE 1)
17. Apply an angular velocity ω_y (Fig. 13)
18. Solve (*Solution-> Solve-> Current LS*)
19. Read results (*Main Menu>General Postproc>Read Results>First Set*). Plot and save as image a contour map of stress in Z direction (*General Postproc>Plot Results>Contour Plot>Nodal Solu> Z- Component of Stress*) (Fig. 14).
20. Choose modal analysis and define options. Include prestress effects (Fig. 15).
21. Read results of modal analysis (*Main Menu> General Postproc> Read Results> By Pick*) and fill Table 4.

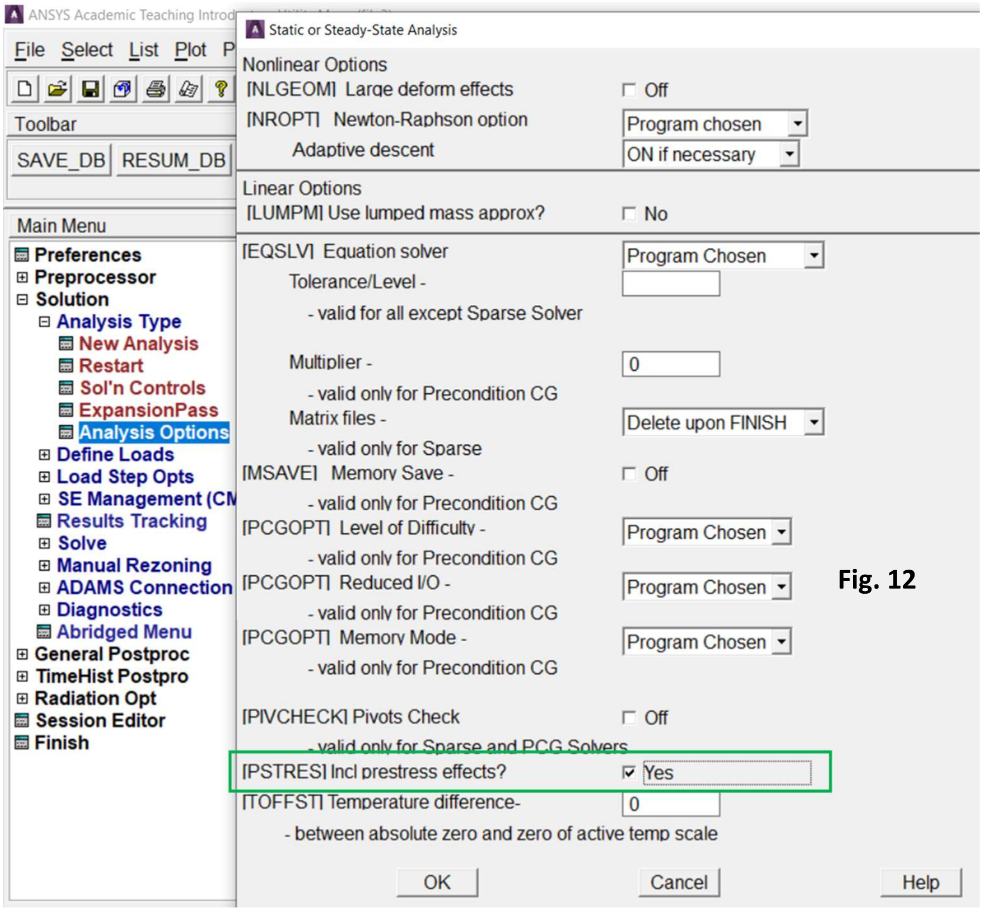


Fig. 12

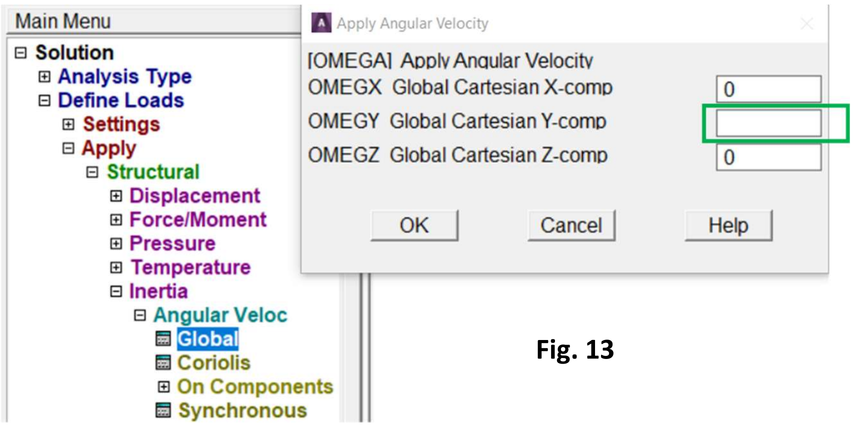
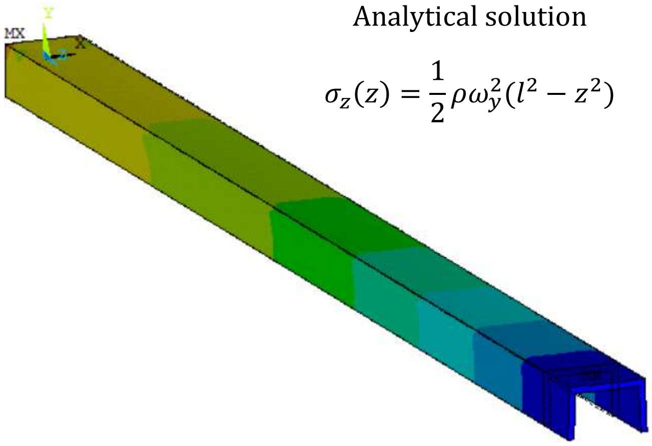


Fig. 13



Analytical solution

$$\sigma_z(z) = \frac{1}{2}\rho\omega_y^2(l^2 - z^2)$$

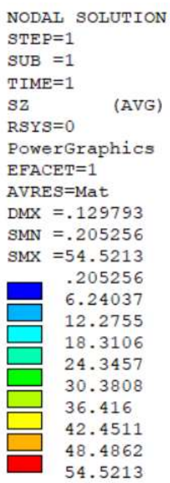


Fig. 14

(l = 1000 mm, ω_y = 100 1/s)

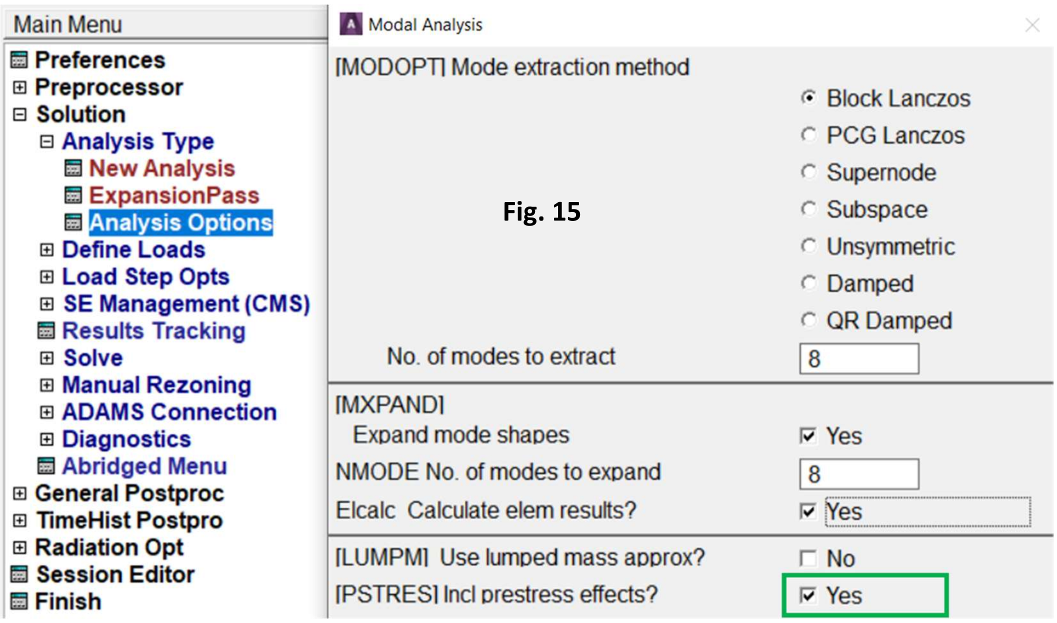


Fig. 15

Discuss results of cases 1-4 and write conclusions.