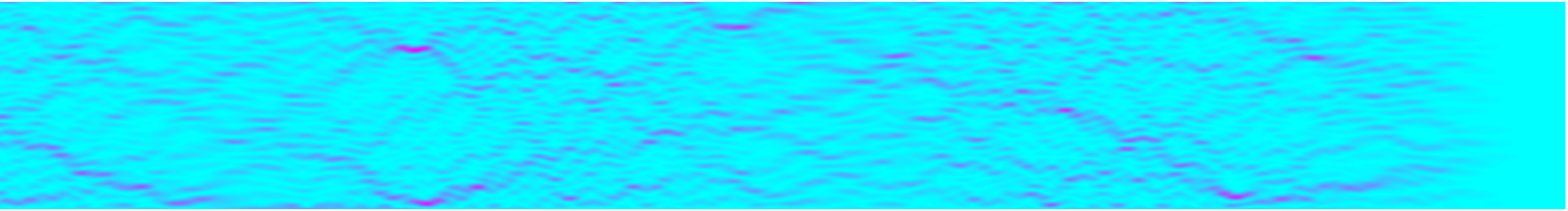
Waves over nonconstant bathymetry. High energy paths in the linear regime.

A. P. Anglart[1,4]

in collaboration with T. Humbert[2], P. Petitjeans[1], V. Pagneux[2], A. Maurel[3]



[1]École Supérieure de Physique et de Chimie Industrielles de la ville de Paris,
Laboratoire de Physqiue et Mécanique des Millieux Hétérogènes, Paris, France

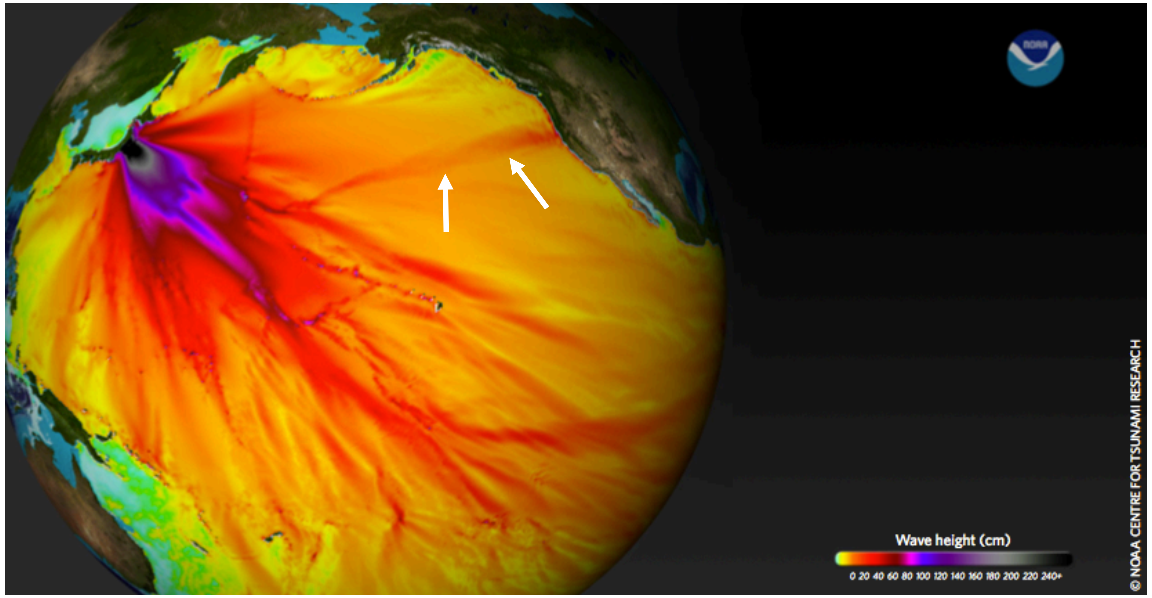
[2]Laboratoire d'Acoustique de l'Université du Maine, Le Mans, France

[3]Institut Langevin LOA, Paris, France

[4]Warsaw University of Technology,
The Faculty of Power and Aeronautical Engineering, Warsaw, Poland

Surface water waves

High energy waves



Branched flow seen in the wave energy map produced after the 2011 Sendai earthquake in Japan. High energy path heading for Crescent City in northern California.
National Oceanic and Atmospheric Administration (2011)

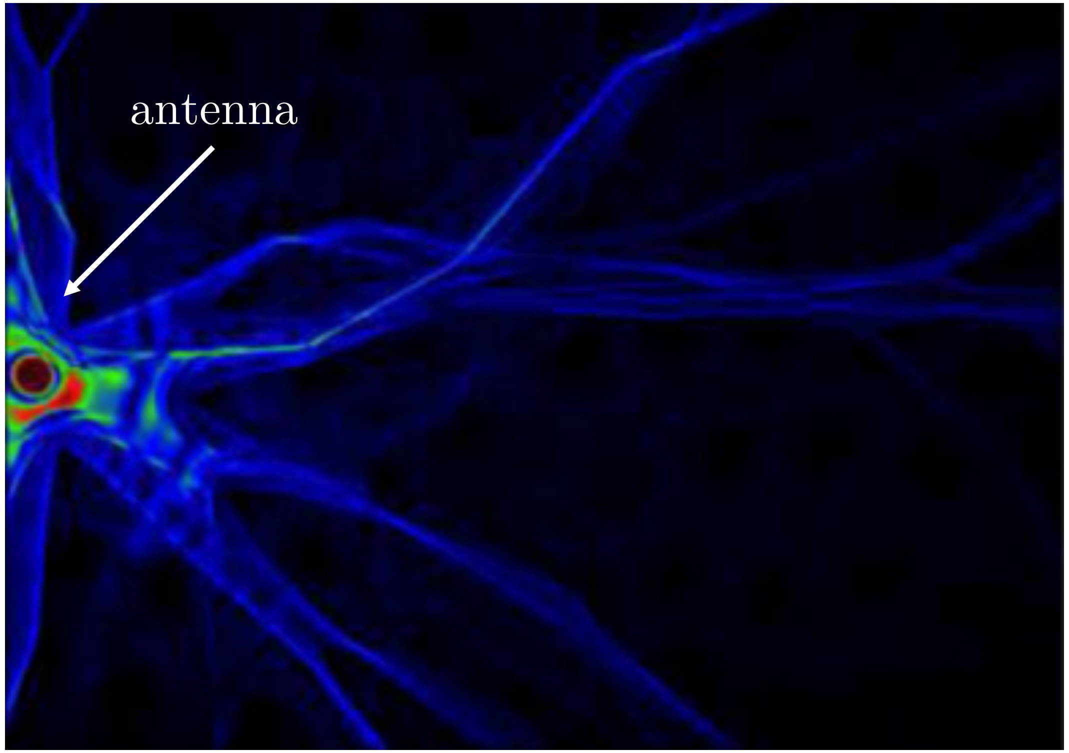
Intensity of a plane wave propagating from left to right in a random bathymetry
Degueldre *et al.* Nature Physics 12 (2016)

Shallow-water waves

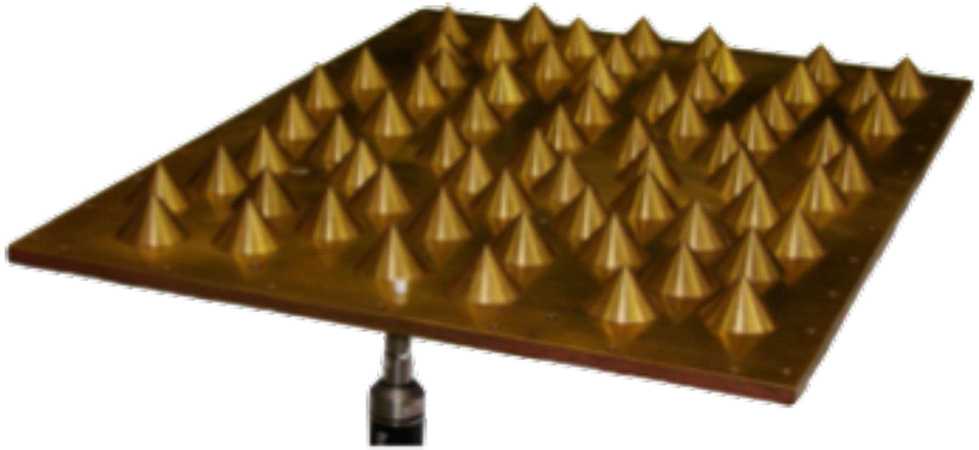
very sensitive to small fluctuations of the bottom topography

High energy paths

Microwaves experiment



Microwave pattern at a frequency f = 30.95 Hz.
Höhman *et al.* 2010, Phys. Rev. Lett. 104 (2010)



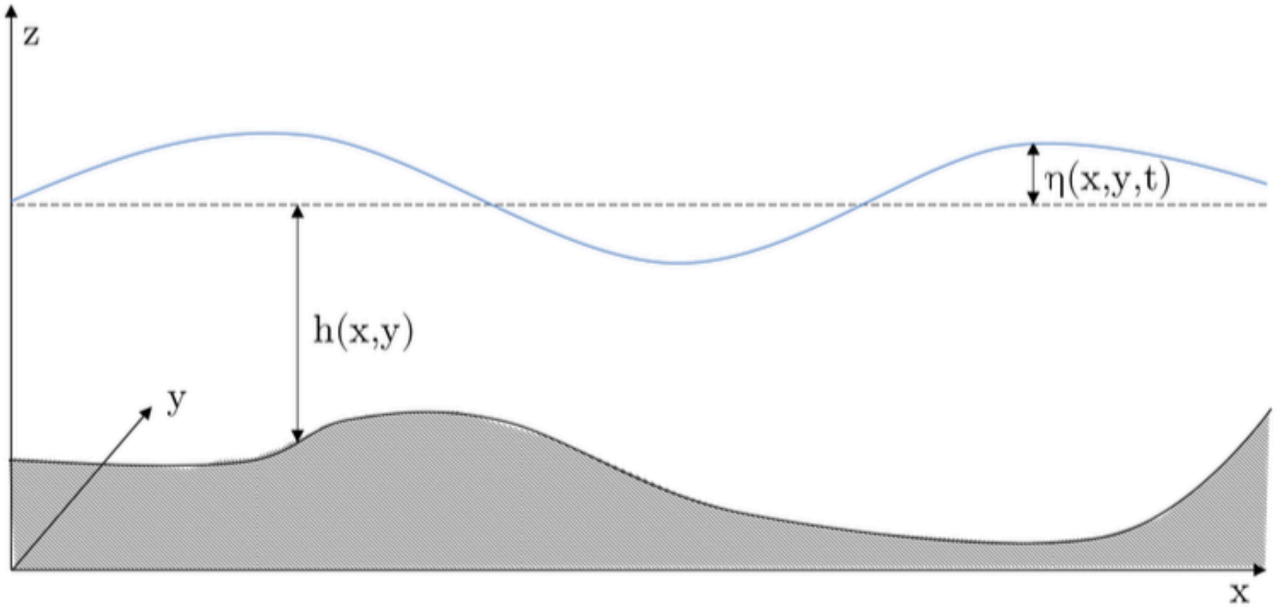
Randomly distributed conical scatterers.
Höhman *et al.* 2010, Phys. Rev. Lett. 104 (2010)

No experimental results for surface water waves so far

Outline

Numerical simulations

Shallow-water equations



1 Linearized shallow-water equation in time domain

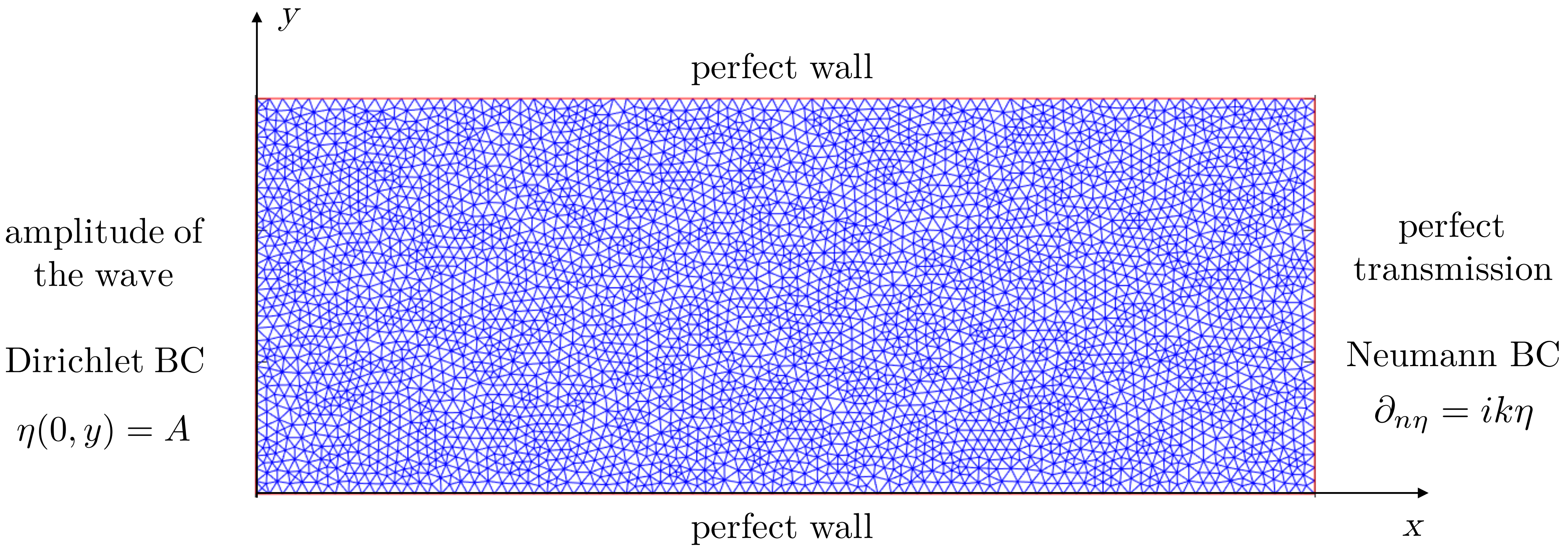
$$\frac{\partial^2}{\partial t^2}\eta + R\frac{\partial}{\partial t}\eta - \nabla(gh(x,y)\nabla\eta) = 0$$

2 Linearized shallow-water equation in frequency domain (complex solution)

$$\nabla(h(x,y)\nabla\eta) + (\frac{\omega^2}{g} - i\omega R^*)\eta = 0$$

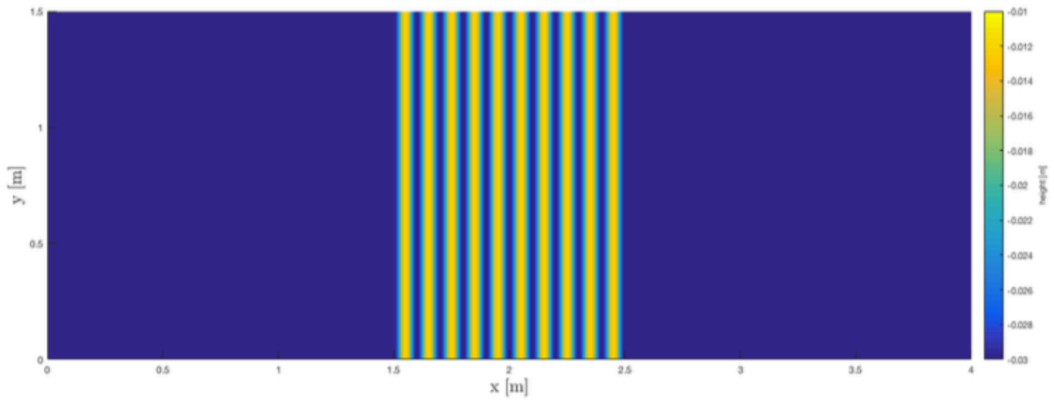
Final element method

Shallow-water equations



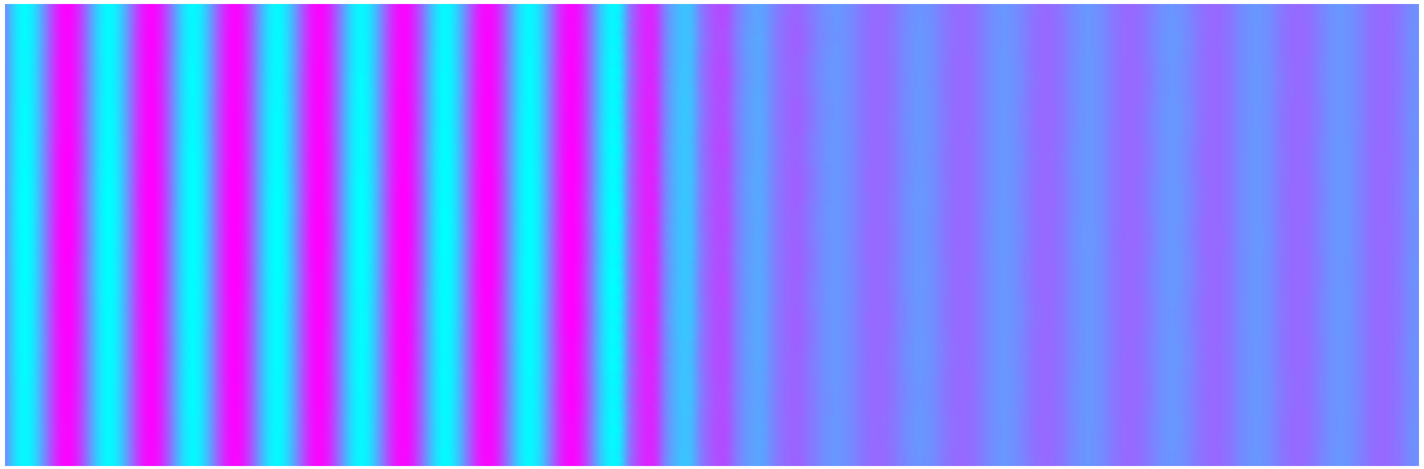
Waves over periodic bathymetry

Bragg's law



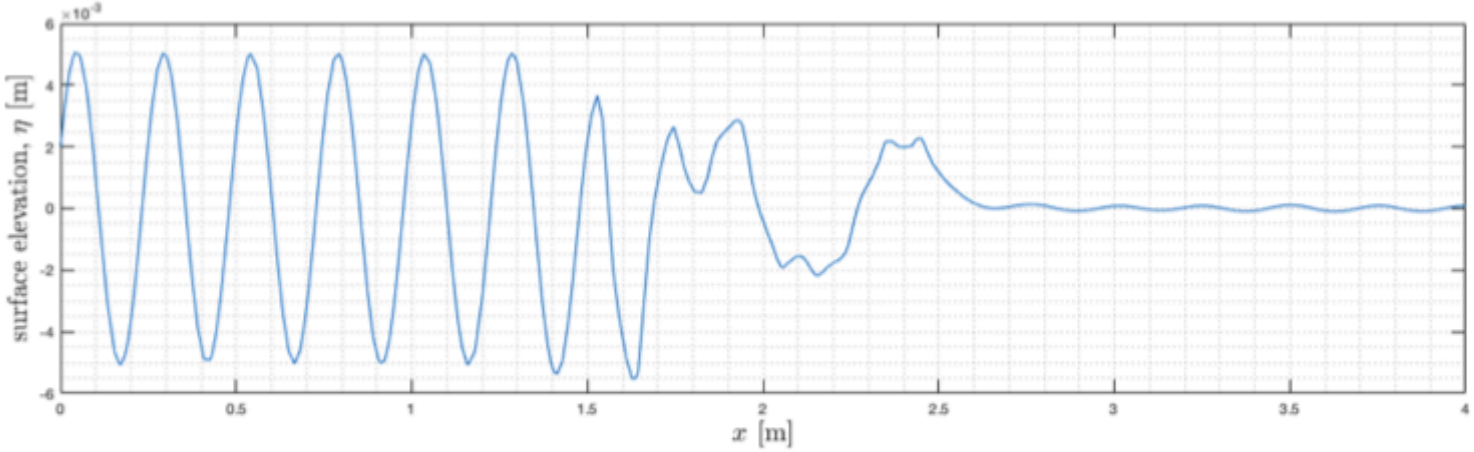
1 Shallow-water equation

$$\nabla(h\nabla\eta) + \frac{\omega^2}{g}\eta = 0$$



2 Bragg's law

$$\lambda = \frac{2d\sin\theta}{n}$$



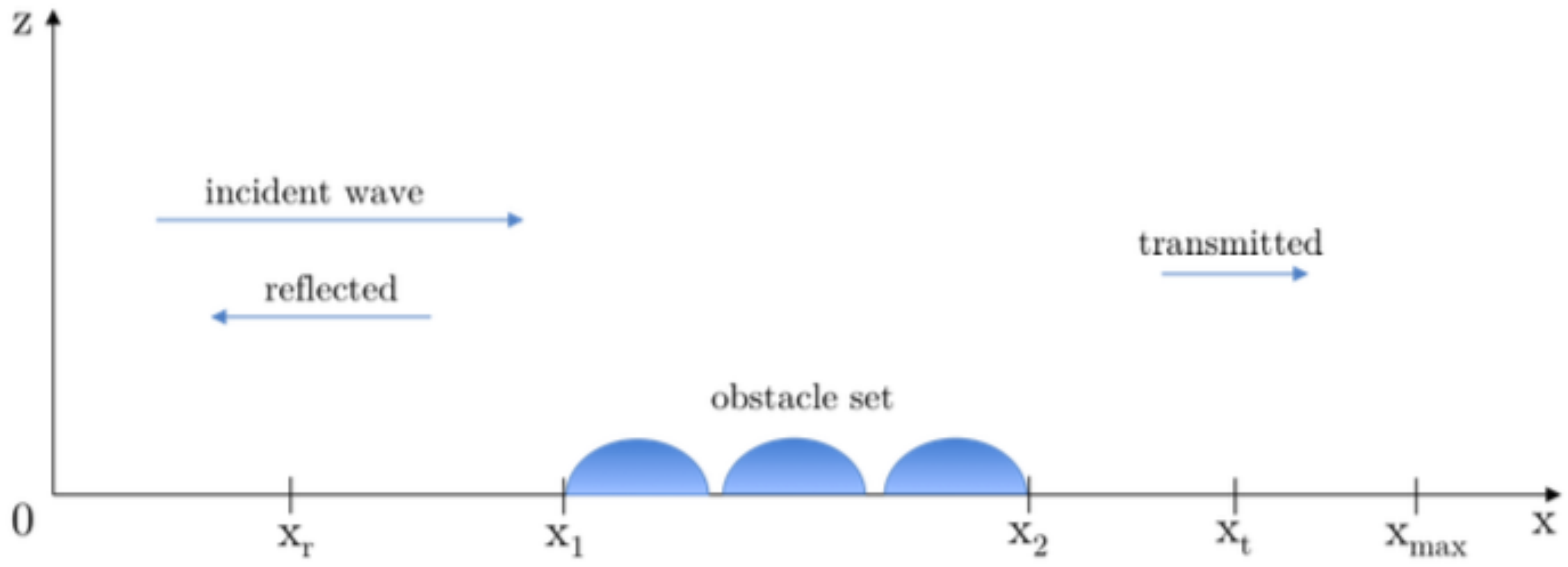
3 Dispersion relation

$$\omega^2 = (gk + \frac{\gamma k^3}{\rho})\tanh(kh)$$

$$f_1 = \frac{\sqrt{gh}}{\lambda} = \frac{\sqrt{gh}}{d} \approx 2.7\text{Hz}$$

Waves over periodic bathymetry

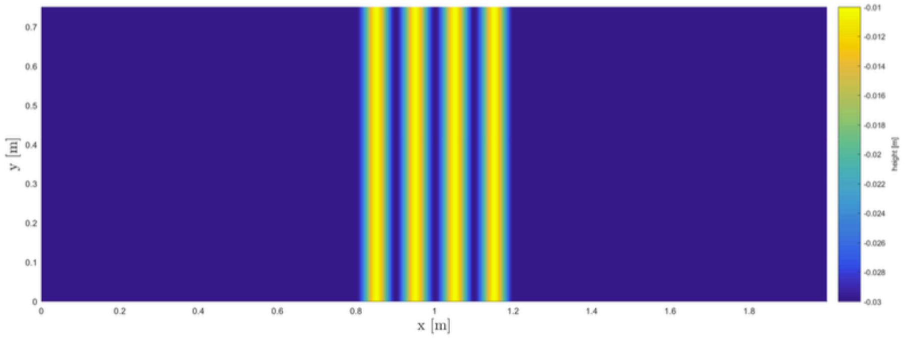
Form of the solution



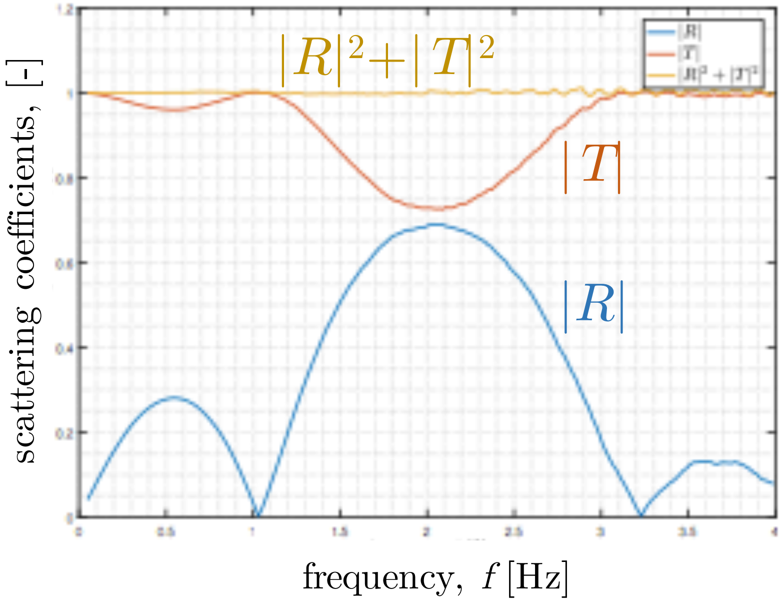
$$\eta(x) = \begin{cases} ae^{-ikx} + Rae^{ikx}, & \text{if } x \in [0, x_1] \\ Tae^{-ikx}, & \text{if } x \in [x_2, x_{max}] \end{cases} \qquad R = -\frac{e^{-ikx_{r_1}} - H_r e^{-ikx_{r_2}}}{e^{ikx_{r_1}} - H_r e^{ikx_{r_2}}} \qquad T = -\frac{e^{-ikx_r} - Re^{ikx_r}}{H_t e^{-ikx_t}}$$

Waves over periodic bathymetry

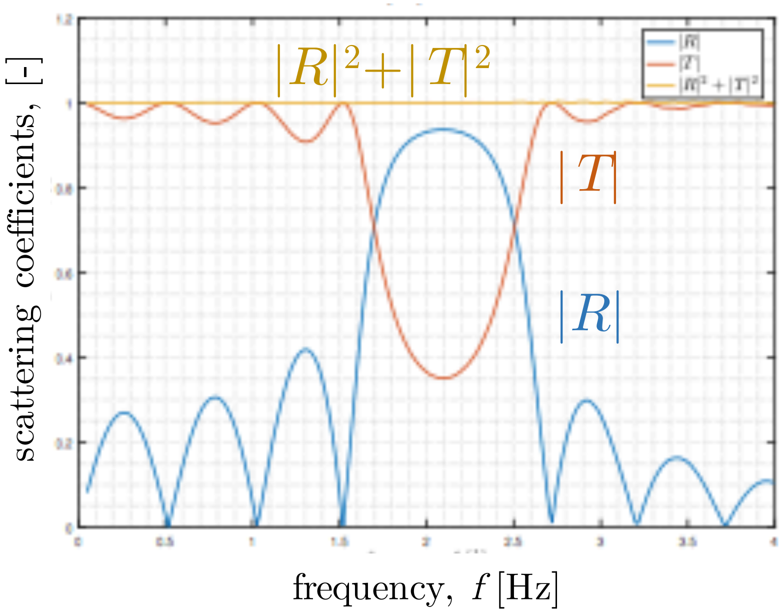
Reflection and transmission coefficients for sinusoidal bars



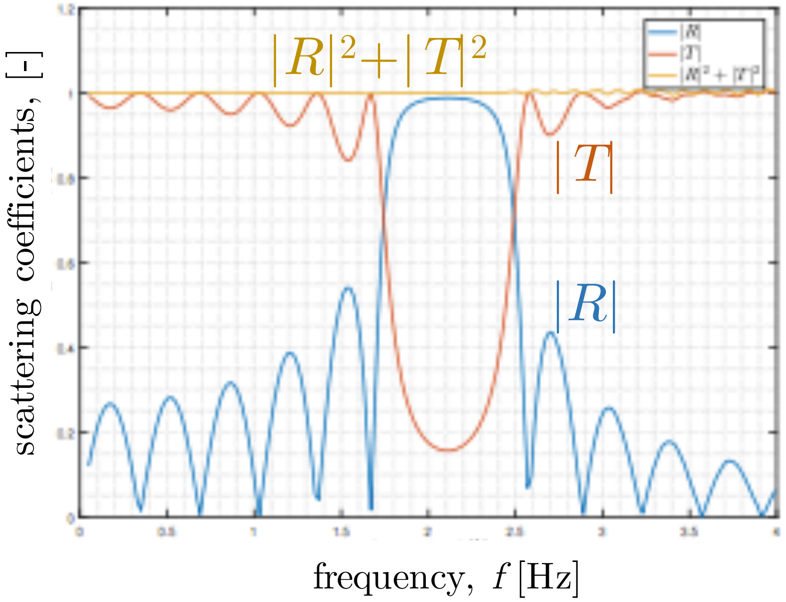
(a)



2 bars



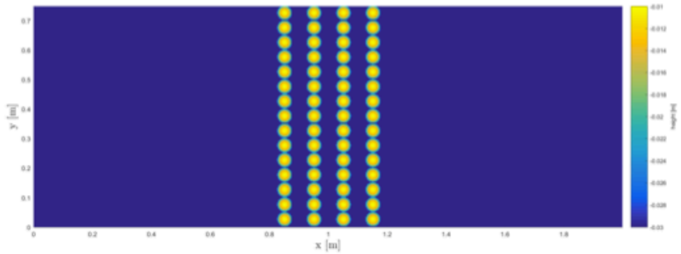
4 bars

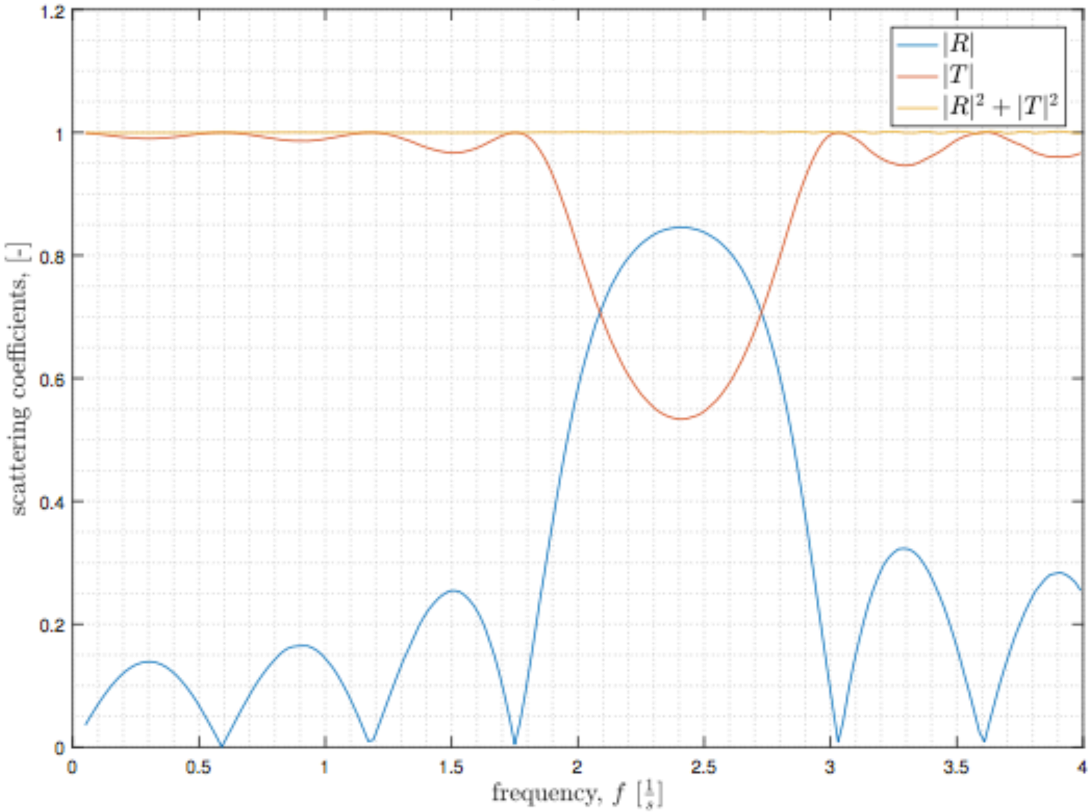


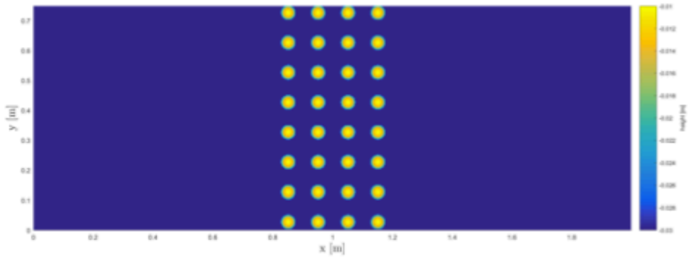
6 bars

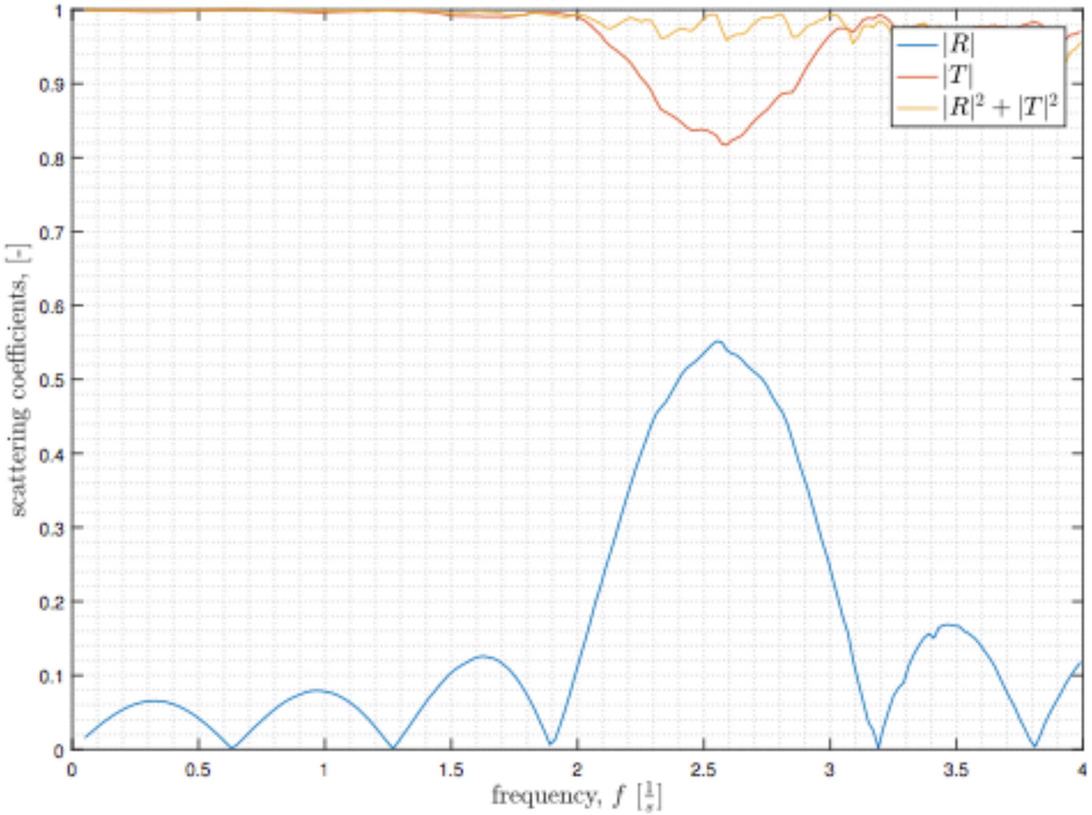
Waves over periodic bathymetry

Reflection and transmission coefficients for hemiellipsoid obstacles



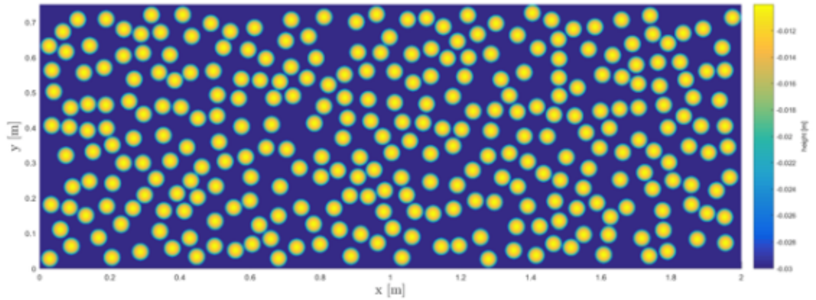




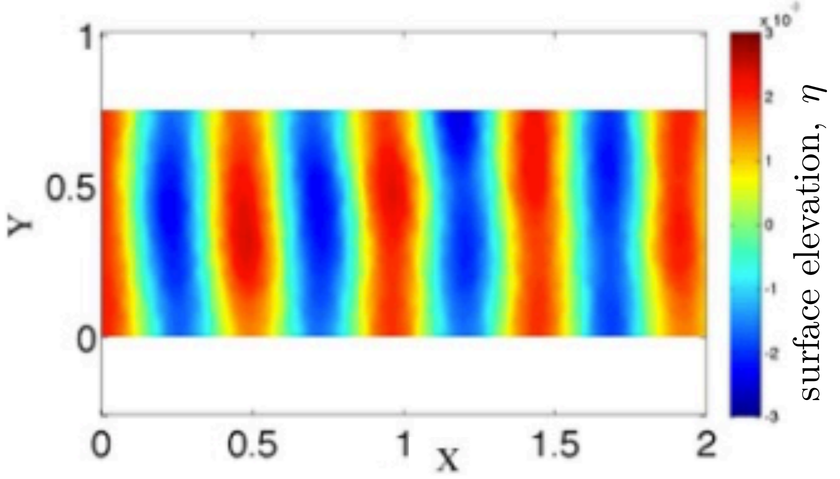


Waves over disordered bathymetry

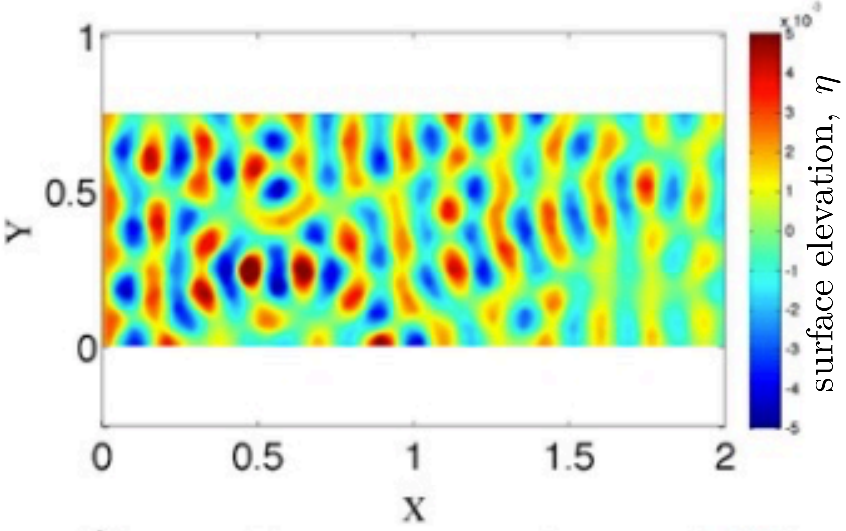
Branching patterns



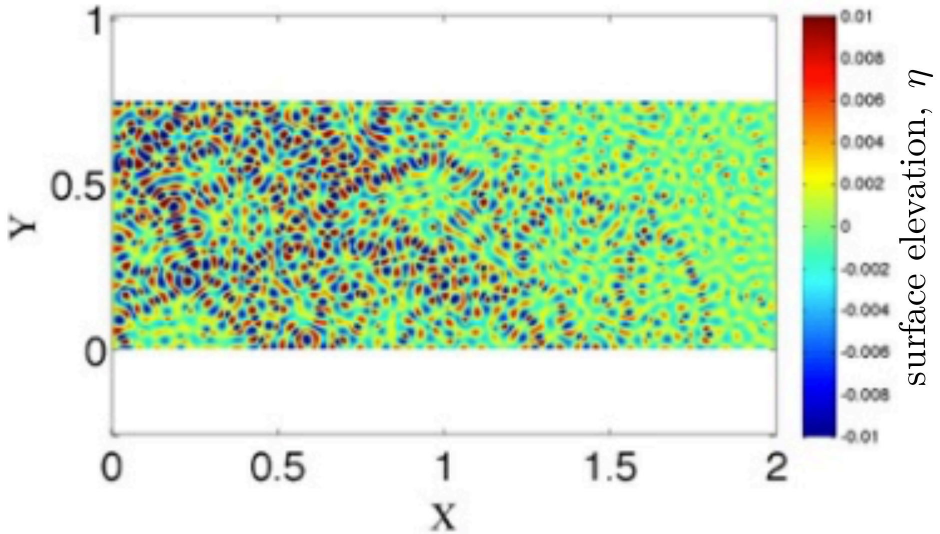
1 Hz



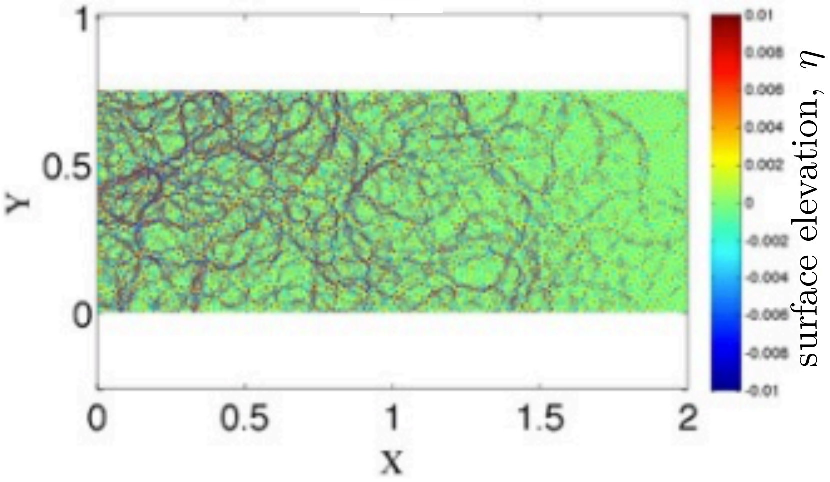
3 Hz



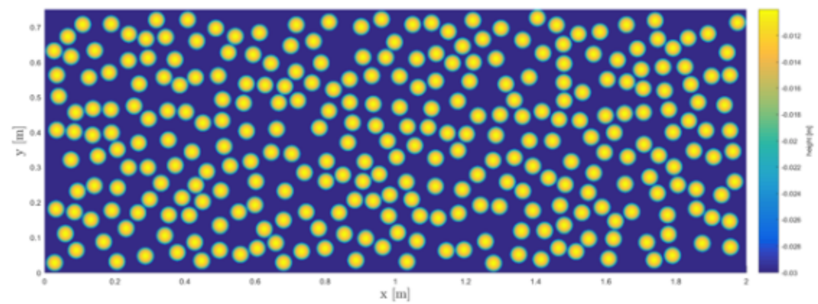
11 Hz

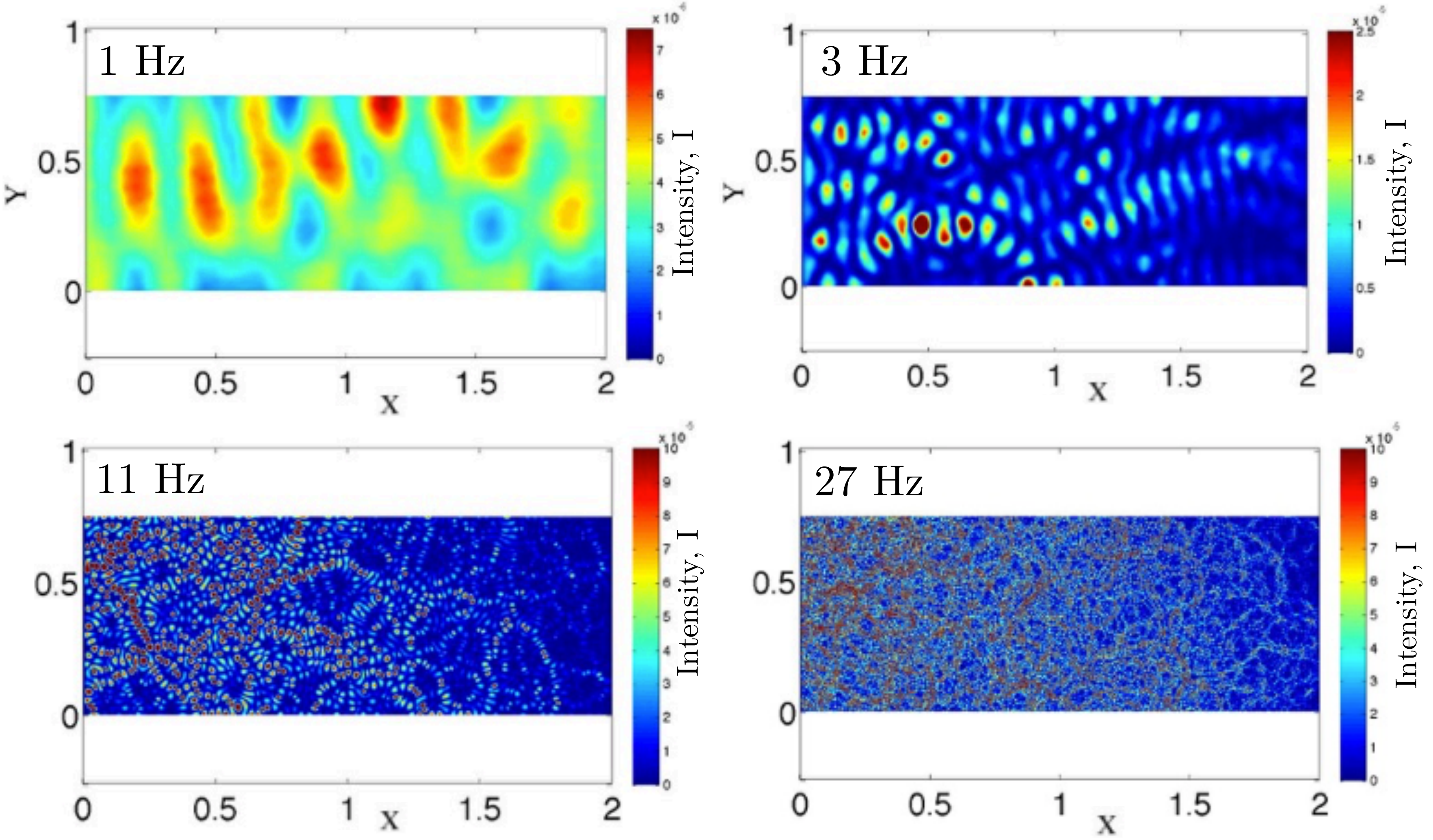


27 Hz



Waves over disordered bathymetry
Intensity maps

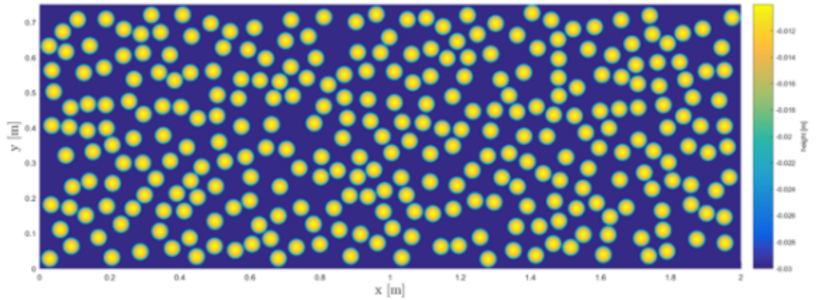




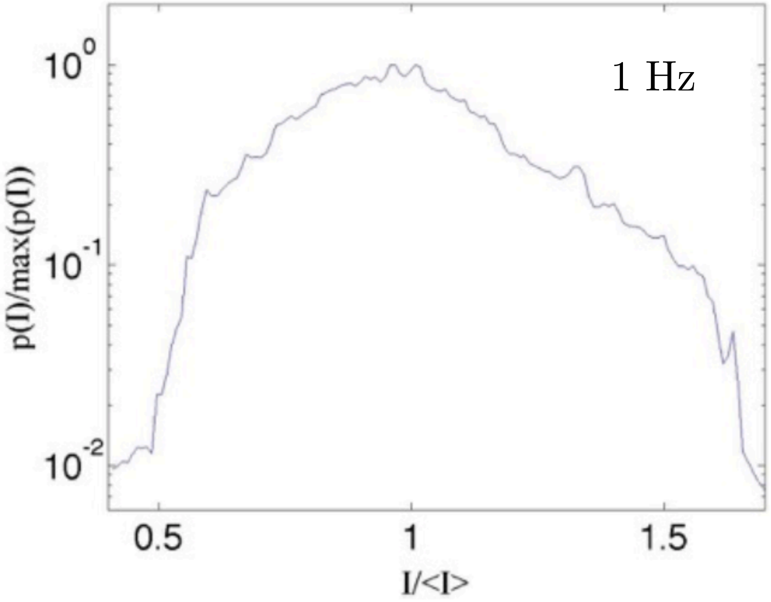
$$\text{Energy} \propto \text{Intensity}$$
$$\text{E} \propto \text{I} = |\eta|^2 + |\nabla\eta|^2$$

Waves over disordered bathymetry

Statistical analysis | Probability density function

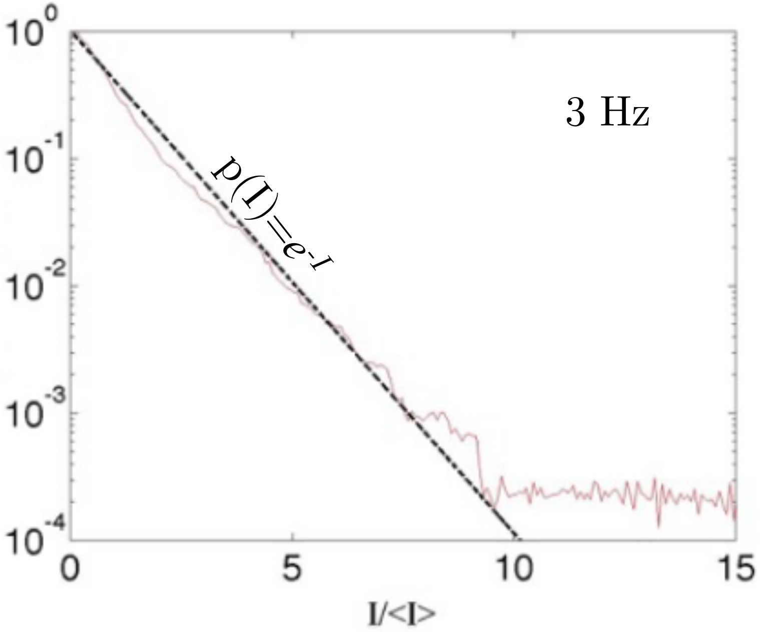


1st regime



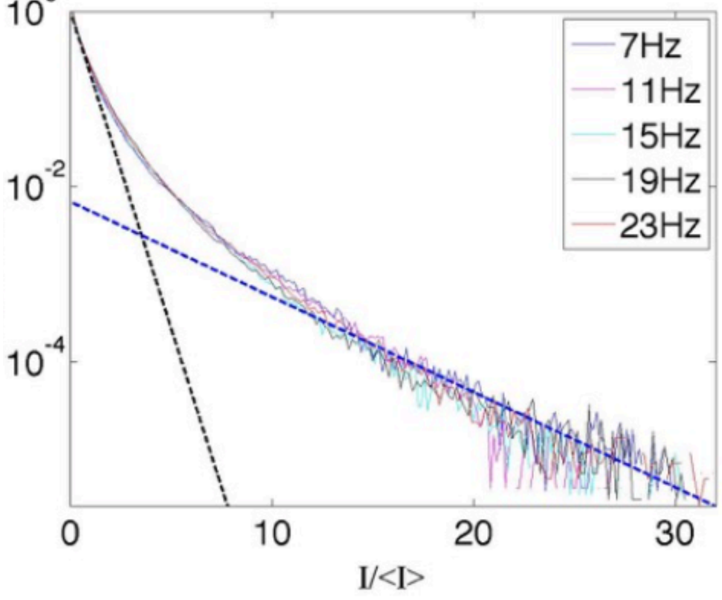
Gaussian distribution

2nd regime



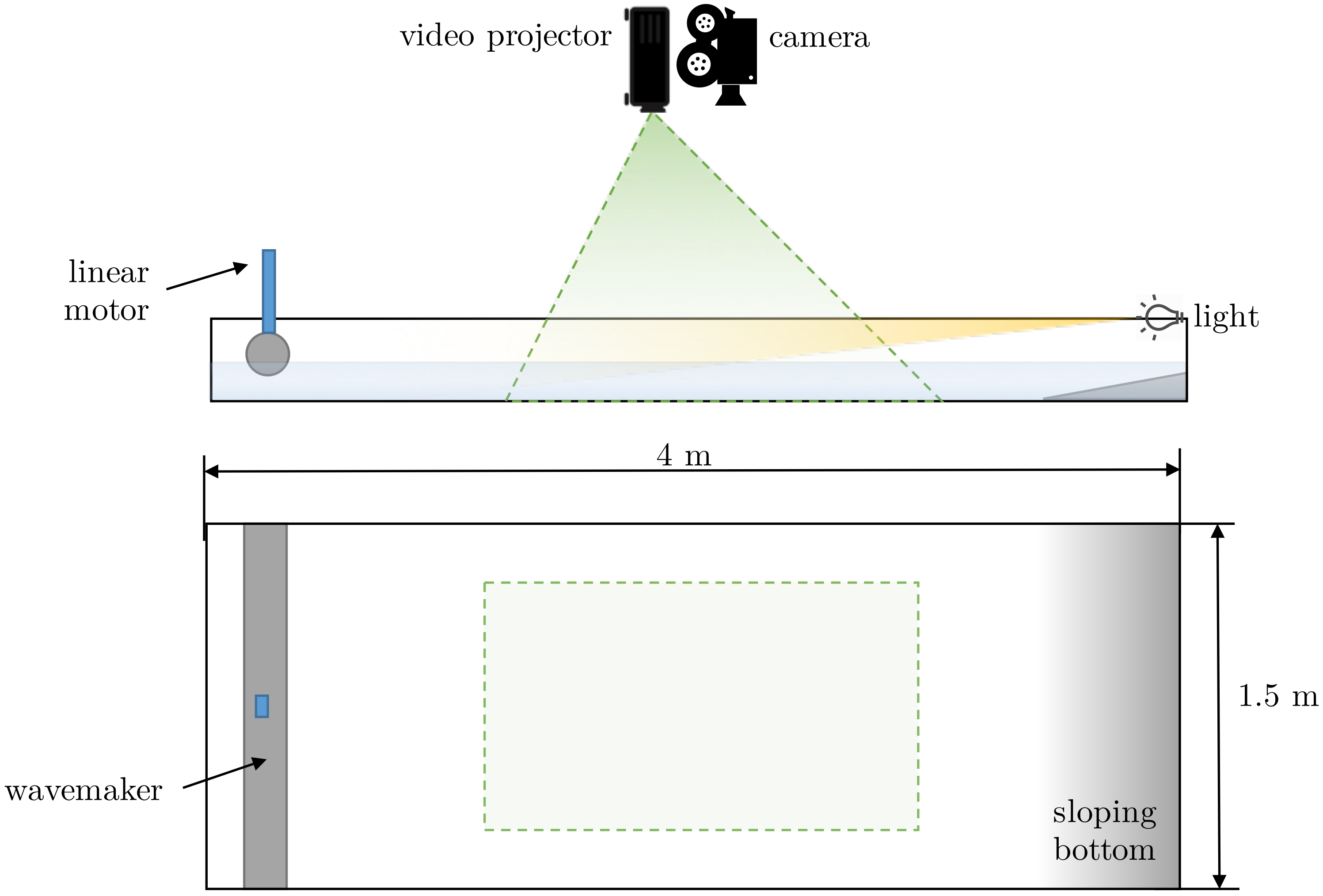
Rayleigh distribution
multiple scattering

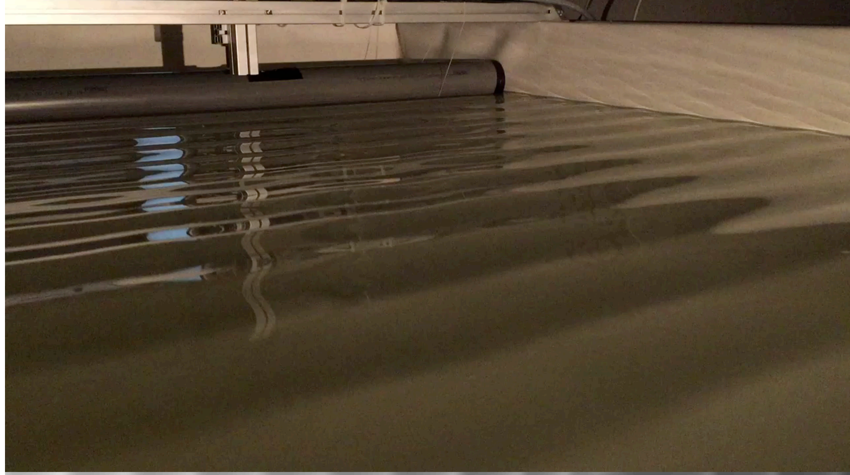
3rd regime

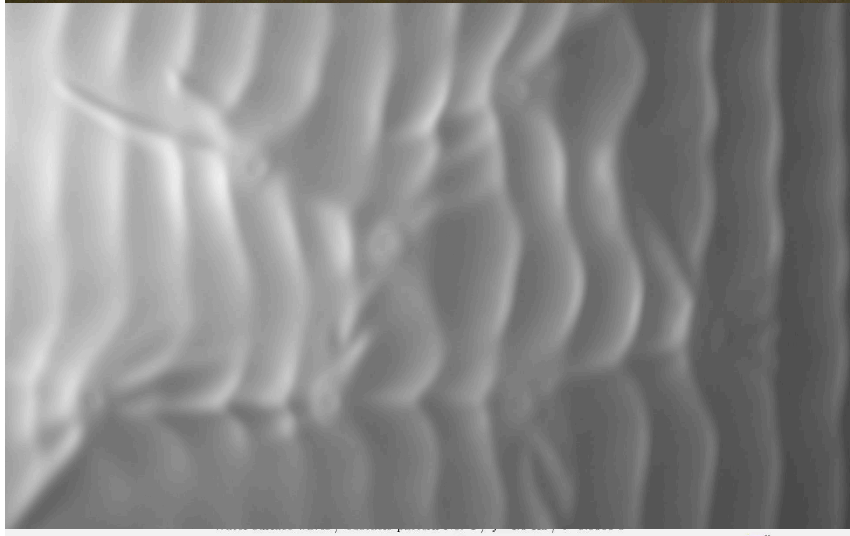


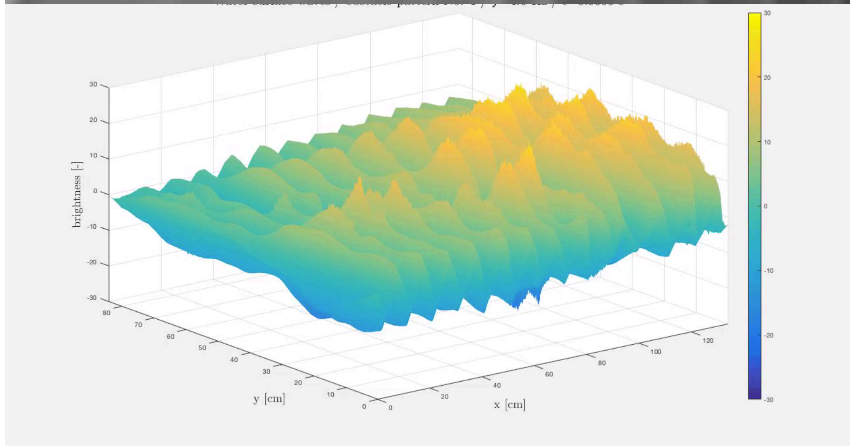
branching patterns

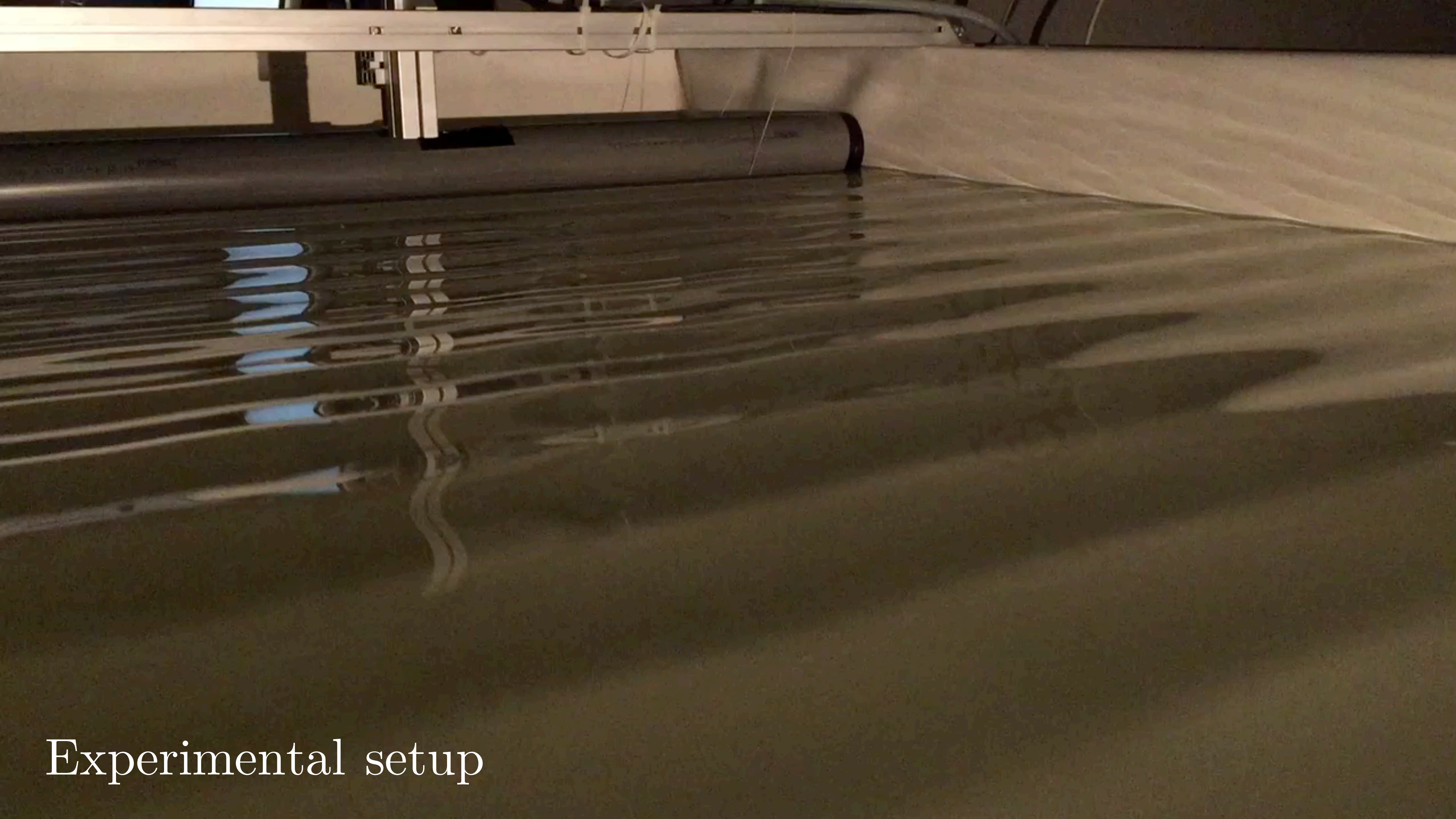
Experimental setup







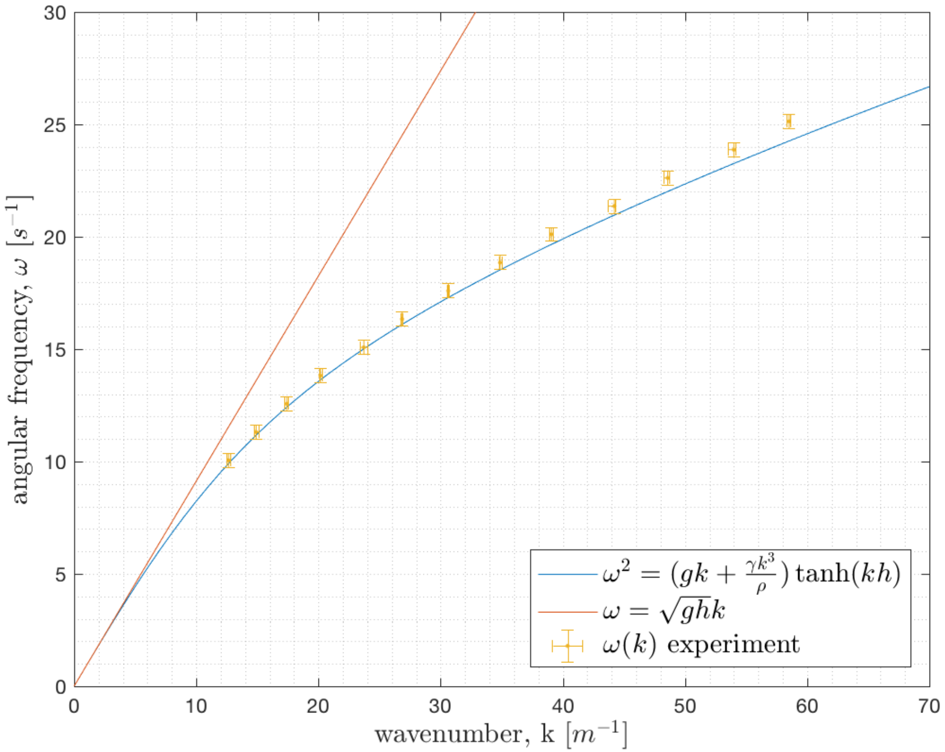



Experimental setup

Waves over a flat bottom

Dispersion relation for water surface waes

Wave propagation for a flat bottom and the frequency f = 2.8 Hz

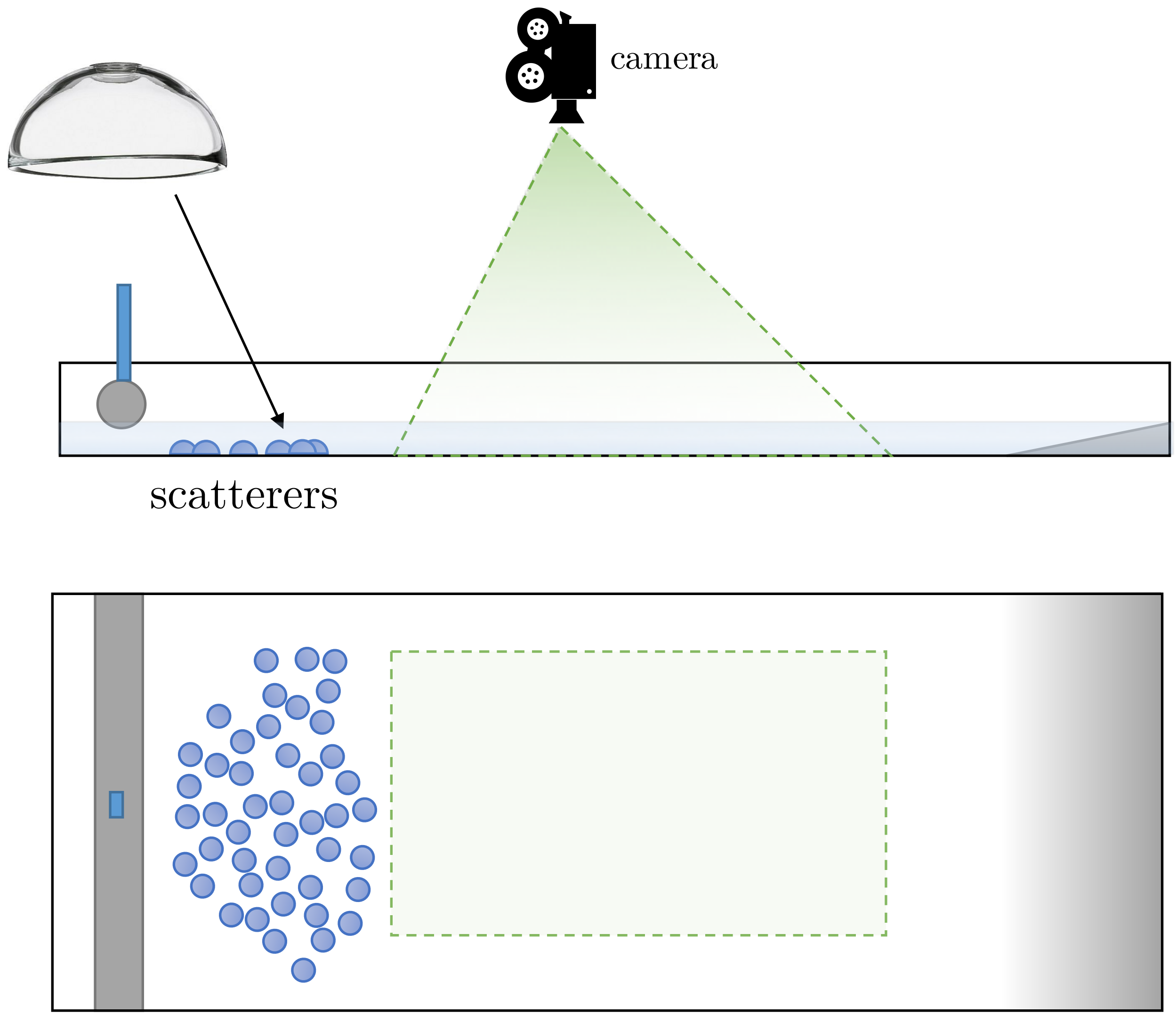


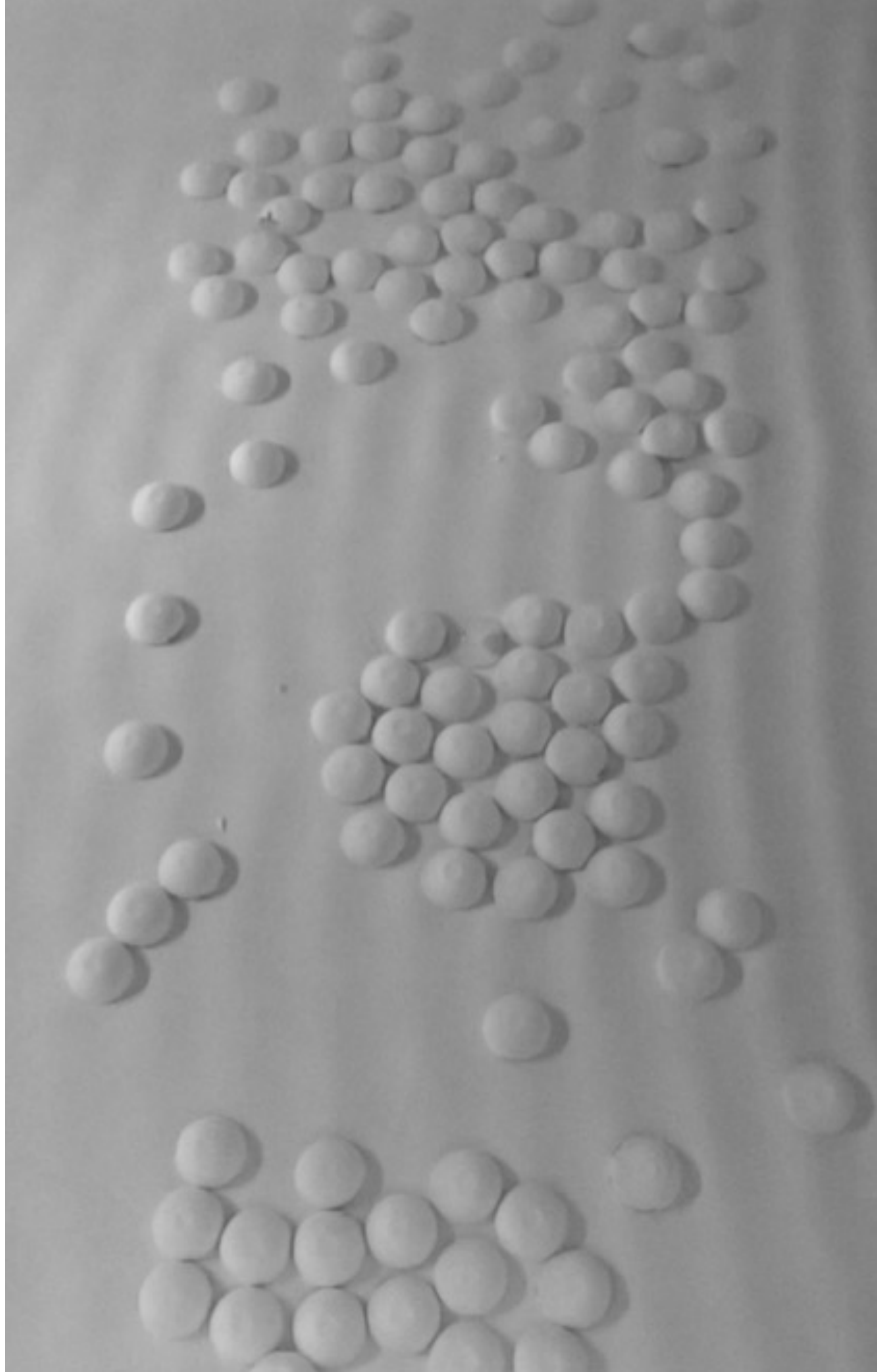
Dispersion relation for water surface waves.

$$\omega^2 = (gk + \frac{\gamma k^3}{\rho})\tanh(kh)$$

Experimental setup

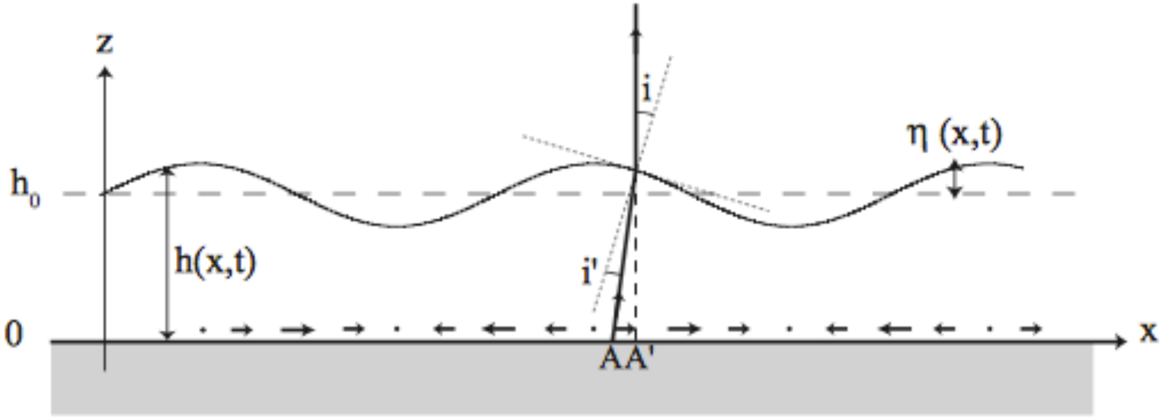
Disordered bathymetry





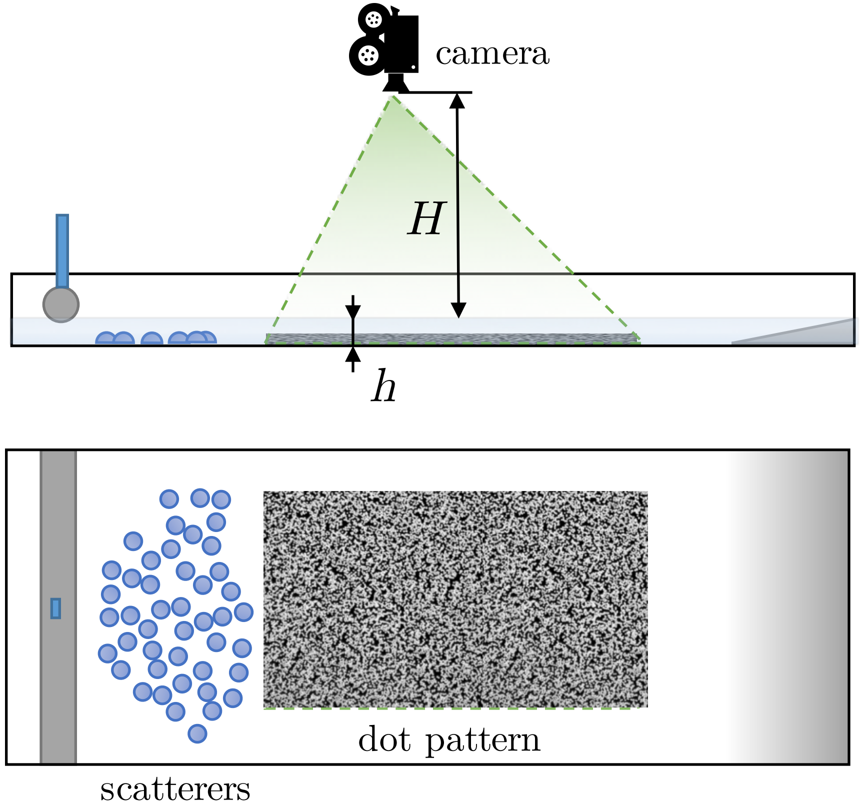
Measurment method

Free-surface synthetic Schlieren



Mesure de la déformation d'une surface libre par analys du déplacement apparent d'un motif aléatoire de points
Moisy *et al.* 18ème Congrés Français de Mécanique (2007)

resolution of ~ 10^{-2} mm



$$\nabla \eta = -\frac{\delta \mathbf{r}}{h^*}, \text{where } \frac{1}{h^*} = \frac{1}{\alpha h} - \frac{1}{H}$$

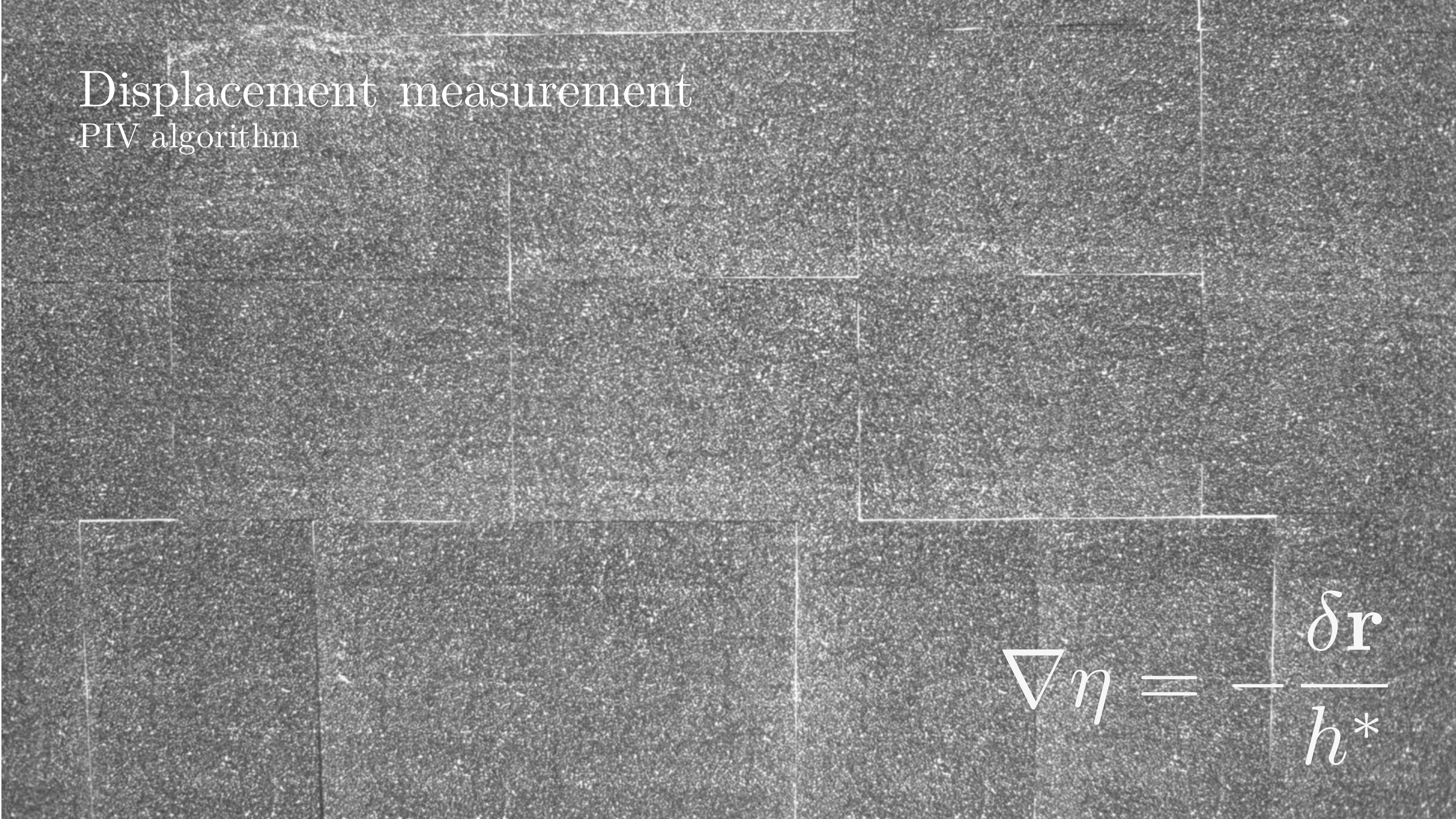
$\delta \mathbf{r}$ - optical displacement field

η - free-surface elevation

α - refraction coefficient (0.24 for air-water interface)

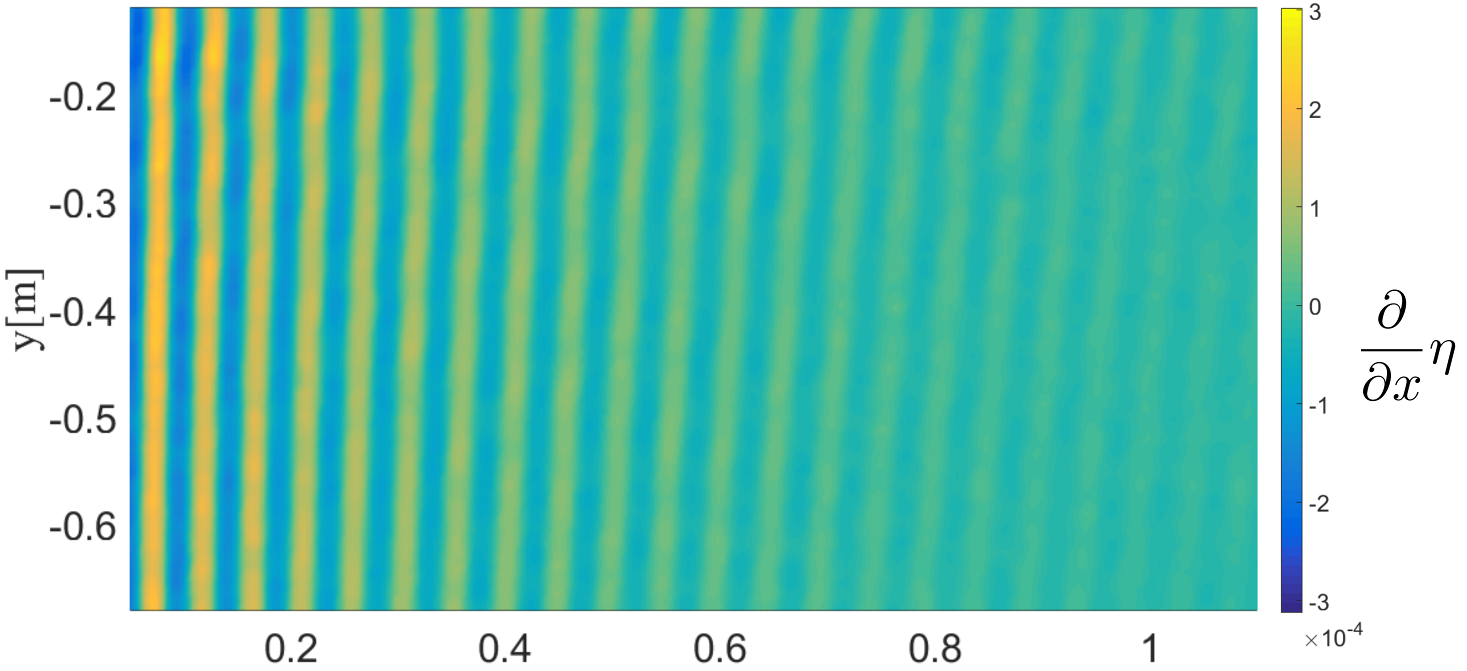
Displacement measurement

PIV algorithm

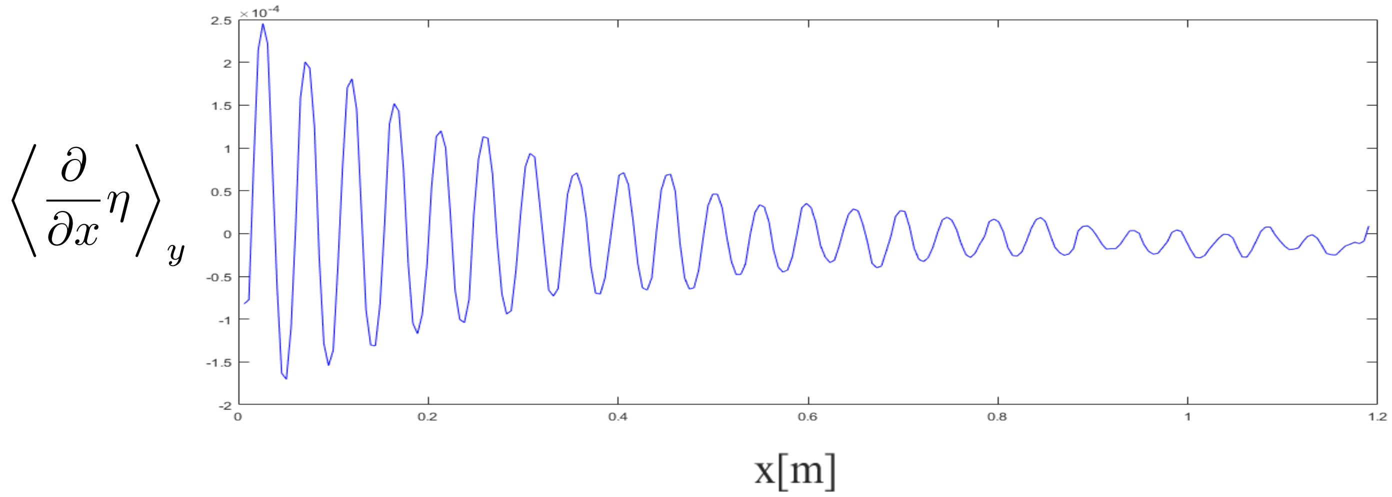


$$\nabla \eta = -\frac{\delta \mathbf{r}}{h^*}$$

First tests



By integrating $\nabla\eta$ we obtain a scalar field of surface eleavation $\eta(x,y)$



resolution of ~ 10^{-2} mm

Conclusion

- numerical smulation has been done to obtain suitable parametres for the experiment
    - specified range of frequency, where branched flow can be observed
- experimental setup built
    - improving the measurement method
    - upgrading the wavemaker to let us acquire needed regime of frequency
- verification of numerical results