# CS-II LAB 5

## Solving a system of ODE's with the 4th-order Runge-Kutta method

Write a C code which solves the system of the ordinary differential equations describing the motion of the cannon ball (of radius $R$ and mass $m$) shot with the initial velocity $V_0$, at the height $h$ above the water level ($\alpha_0$ denotes the shot angle). The motion of the ball is governed by the following system of the ODE's and the initial conditions

$$m\frac{d^2x}{dt^2} = -C_D S\frac{\rho}{2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\frac{dx}{dt}$$

$$m\frac{d^2y}{dt^2} = -C_D S\frac{\rho}{2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\frac{dy}{dt} - mg$$

$$x(t_0)=0 \quad , \quad y(t_0)=h$$

$$\frac{dx}{dt}(t_0)=V_0\cos\alpha_0 \quad , \quad \frac{dy}{dt}(t_0)=V_0\sin\alpha_0$$

where:

***g*** – acceleration due to gravity (9,81 $\frac{m}{s^2}$),

**ρ** – density of the medium (water - 1000 $\frac{kg}{m^3}$, air - 1,2 $\frac{kg}{m^3}$),

$\boldsymbol{S=\pi R^2}$ – ball's reference area,

$\boldsymbol{C_D}$– drag coefficient .

Derive an equivalent system of four 1st order differential equations . This system can be view as a particular example of a general one, define as follows

$$\mathbf{z}'(t) = \mathbf{F}(t,\mathbf{z}(t)) \quad , \quad \text{where}$$

$$\mathbf{z}(t) = [z_1(t),..,z_n(t)]^T,$$

$$\mathbf{F}(t,\mathbf{z}(t)) = [F_1,..,F_n]^T(t,z_1(t),..,z_n(t)).$$

Write the function ***flying_ball*** which calculates the vector of the first derivatives, i.e. the vector **F**(t,**z**(t)), corresponding to the obtained differential system (DS). In order to account for a variable density you can write the function where ρ is defined as a piecewise constant function of the coordinate y.

Write the routine ***rk4_step*** which performs a single integrations step of the general DS, using the 4th-order RK method:

$$\mathbf{z}(t+\Delta t) = \mathbf{z}(t) + (\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)/6,$$

$$\mathbf{k}_1 = \Delta t \cdot \mathbf{F}(t,\mathbf{z}(t)),$$

$$\mathbf{k}_2 = \Delta t \cdot \mathbf{F}(t+\Delta t/2, \mathbf{z}(t)+\mathbf{k}_1/2),$$

$$\mathbf{k}_3 = \Delta t \cdot \mathbf{F}(t+\Delta t/2, \mathbf{z}(t)+\mathbf{k}_2/2),$$

$$\mathbf{k}_4 = \Delta t \cdot \mathbf{F}(t+\Delta t, \mathbf{z}(t)+\mathbf{k}_3).$$

**Note**: in order to obtain a really universal tool, the list of the arguments of ***rk4_step*** should contain a pointer to a user-defined function which computes the derivatives (such as the ***flying_ball*** function).

Write the main function which reads from the keyboard a value of the time step and the number of steps (or the integration time). Design the integration loop, where the routine ***rk4_step*** is used at each cycle. Dump the current value of the time variable t and the four components of the vector **z**(t) to a disk output file at each cycle of the loop.

Run sample calculations for m = 5 kg, S = 0.008 $m^2$, $C_D$ = 0.4, $V_0$=150 m/s, $\alpha$=$20^0$, h=200m. and Δt=0.01. Adjust the number of steps so that the ball sinks in the water under the line y = -h/2.

Use Grapher to plot the ball's trajectory and the velocity components versus time. Repeat the calculations with a different time step (like Δt = 0.005 and 0.02) and make sure that the obtained results are "step-independent".