CS-II LAB 5

Solving a system of ODE's with the 4th-order Runge-Kutta method

Write a C code which solves the system of the ordinary differential equations describing the motion of the pendulum. The motion of the pendulum is defined by the following ODE and initial conditions:

$$\frac{d^2\alpha}{dt^2} = -\frac{g}{l}\sin\alpha$$
$$\alpha(t_0) = \alpha_0$$
$$\frac{d\alpha}{dt}(t_0) = \varpi_0$$

where:

g – acceleration due to gravity ($9{,}81\frac{m}{s^2}$),

l – length of the pendulum,

α – angle defining current position of the pendulum.

Derive an equivalent system of two 1st order differential equations . This system can be view as a particular example of a general one, define as follows

$$\mathbf{z}'(t) = \mathbf{F}(t, \mathbf{z}(t)) \quad , \quad \text{where}$$
$$\mathbf{z}(t) = [z_1(t), .., z_n(t)]^T ,$$
$$\mathbf{F}(t, \mathbf{z}(t)) = [F_1, .., F_n]^T (t, z_1(t), .., z_n(t)).$$

Write the function ***rhs*** which calculates the vector of the first derivatives, i.e. the vector **F**(t,**z**(t)), corresponding to the obtained differential system (DS).

Use the routine ***vrk4*** which can be downloaded from the server. This function implements the 4th-order RK method:

$$\mathbf{z}(t + \Delta t) = \mathbf{z}(t) + (\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)/6,$$
$$\mathbf{k}_1 = \Delta t \cdot \mathbf{F}(t, \mathbf{z}(t)),$$
$$\mathbf{k}_2 = \Delta t \cdot \mathbf{F}(t + \Delta t/2, \mathbf{z}(t) + \mathbf{k}_1/2),$$
$$\mathbf{k}_3 = \Delta t \cdot \mathbf{F}(t + \Delta t/2, \mathbf{z}(t) + \mathbf{k}_2/2),$$
$$\mathbf{k}_4 = \Delta t \cdot \mathbf{F}(t + \Delta t, \mathbf{z}(t) + \mathbf{k}_3).$$

Write the main function which reads from the keyboard a value of the time step and the number of steps (or the integration time). Design the integration loop, where the routine ***vrk4*** is used at each cycle. Dump the current value of the time variable t and the all components of the vector **z**(t) to an output file at each cycle of the loop.

Run sample calculation which computes α(t) and angular velocity dα(t)/dt for 0<t<10s. In addition compute total energy (sum of potential and kinetic energies) of the system as a function of time - E(t). Create plots visualizing computed solution.