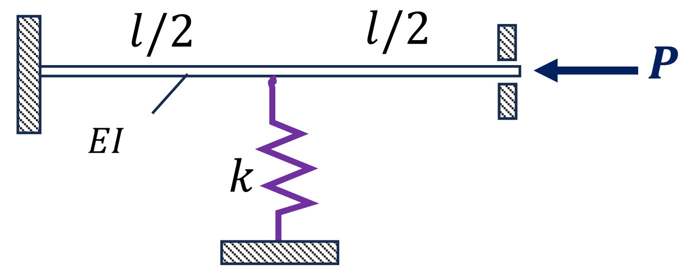
Wydział Mechaniczny Energetyki i Lotnictwa
Zakład Wytrzymałości Materiałów i Konstrukcji

Finite element method 2 (FEM2)

Buckling analysis. Examples

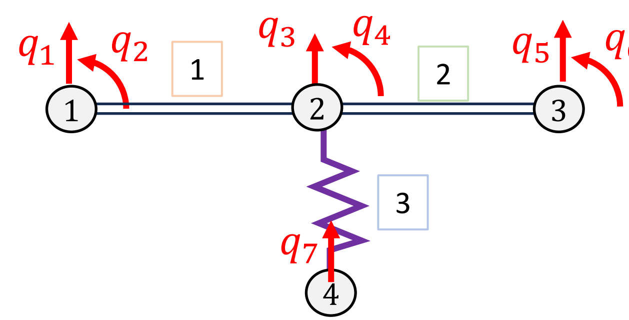
Example 1. Find the critical load for the beam and spring model.



$$([K]-\lambda_*[K_\sigma])\{q\}=0$$

$$[k]_1=[k]_2=\frac{2EI}{l_e^3}\begin{bmatrix}6 & 3l_e & -6 & 3l_e\\ 3l_e & 2l_e^2 & -3l_e & l_e^2\\ -6 & -3l_e & 6 & -3l_e\\ 3l_e & l_e^2 & -3l_e & 2l_e^2\end{bmatrix}$$

$$[k]_3=\frac{2EI}{l_e^3}\begin{bmatrix}\frac{kl_e^3}{2EI} & -\frac{kl_e^3}{2EI}\\ -\frac{kl_e^3}{2EI} & \frac{kl_e^3}{2EI}\end{bmatrix}$$



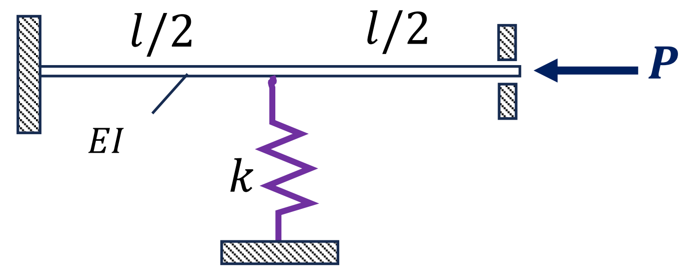
Global stiffness matrix:

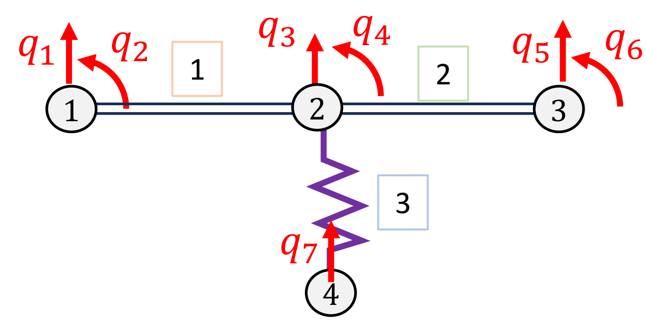
$$[K]=\frac{2EI}{l_e^3}\begin{bmatrix}6 & 3l_e & -6 & 3l_e & & & \\ & 2l_e^2 & -3l_e & l_e^2 & & & \\ & & 6+6+\frac{kl_e^3}{2EI} & 3l_e-3l_e & -6 & 3l_e & -\frac{kl_e^3}{2EI}\\ & & & 2l_e^2+2l_e^2 & -3l_e & l_e^2 & \\ & & & & 6 & -3l_e & \\ & & & & & 2l_e^2 & \\ & & & & & & \frac{kl_e^3}{2EI}\end{bmatrix}$$

Symm.

$$([K]-\lambda_*[K_\sigma])\{q\}=0$$

$$[k_\sigma]_1=[k_\sigma]_2=\frac{1}{30l_e}\begin{bmatrix} 36 & 3l_e & -36 & 3l_e \\ 3l_e & 4l_e^2 & -3l_e & -l_e^2 \\ -36 & -3l_e & 36 & -3l_e \\ 3l_e & -l_e^2 & -3l_e & 4l_e^2 \end{bmatrix}$$





The prestress matrix:

$$[K_\sigma]=\frac{1}{30l_e}\begin{bmatrix} 36 & 3l_e & -36 & 3l_e & & & \\ & 4l_e^2 & -3l_e & -l_e^2 & & & \\ & & 36+36 & 3l_e-3l_e & -36 & 3l_e & \\ & & & 4l_e^2+4l_e^2 & -3l_e & -l_e^2 & \\ & & \text{Symm.} & & 36 & -3l_e & \\ & & & & & 4l_e^2 & \\ & & & & & & \end{bmatrix}$$

Boundary conditions: $q_1 = q_2 = q_5 = q_6 = q_7 = 0$

$$\left(\frac{2EI}{l_e^3}\begin{bmatrix} 12+\frac{kl_e^3}{2EI} & 0 \\ 0 & 4l_e^2 \end{bmatrix} - \frac{\lambda_*}{30l_e}\begin{bmatrix} 72 & 0 \\ 0 & 8l_e^2 \end{bmatrix}\right)\begin{Bmatrix} q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The auxiliary constants:

$$\lambda = \frac{l_e^2}{60EI}\cdot\lambda_* \qquad \beta = \frac{kl_e^3}{2EI}$$

$$\begin{bmatrix} 12+\beta-72\lambda & 0 \\ 0 & 4l_e^2(1-2\lambda) \end{bmatrix}\begin{Bmatrix} q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The determinant is zero if:

$$(12+\beta-72\lambda)(1-2\lambda) = 0$$

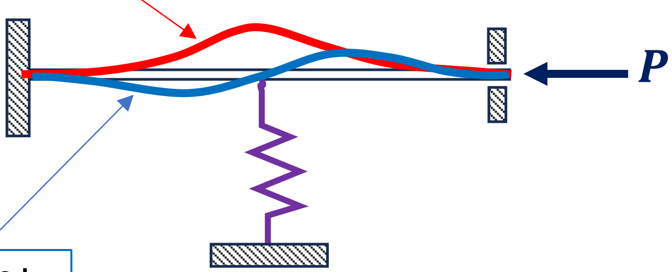
roots: $\lambda_1 = \frac{12+\beta}{72}$ $\quad \lambda_2 = \frac{1}{2}$

If $\beta < 24 \rightarrow k < \frac{48EI}{l_e^3} \rightarrow \lambda_1 < \lambda_2$ (a weak spring)

results in **the first buckling mode**:

$$\lambda_1^* = \frac{60EI}{l_e^2}\lambda_1 = \left(40 + \frac{20}{6}\beta\right)\frac{EI}{l_e^2}$$

$$\lfloor q \rfloor_1 = \lfloor 0,0,q_3,0,0,0,0 \rfloor$$



If $\beta > 24 \rightarrow k > \frac{48EI}{l_e^3} \rightarrow \lambda_2 < \lambda_1$ (a stiff spring)

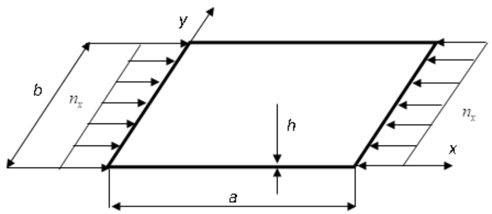
and **the second buckling mode** occures:

$$\lambda_2^* = \frac{60EI}{l_e^2}\lambda_2 = \frac{30EI}{l_e^2}$$

$$\lfloor q \rfloor_2 = \lfloor 0,0,0,q_4,0,0,0 \rfloor$$

Example 2. Compression of a plate hinged at the edges (uz=0)

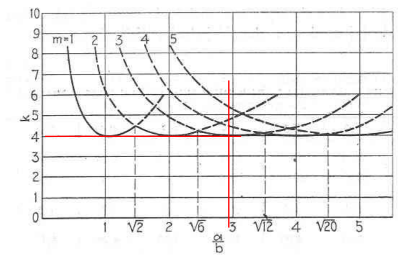
a= 885mm, b= 302mm, h = 2.5mm, E=70000 MPa, ν=0.33

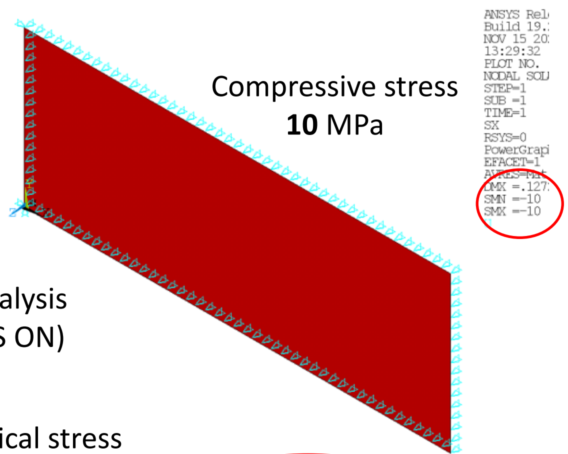


$$\sigma_{cr} = k\frac{\pi^2}{12(1-\nu^2)}\frac{Eh^2}{b^2}$$

(theoretical solution)

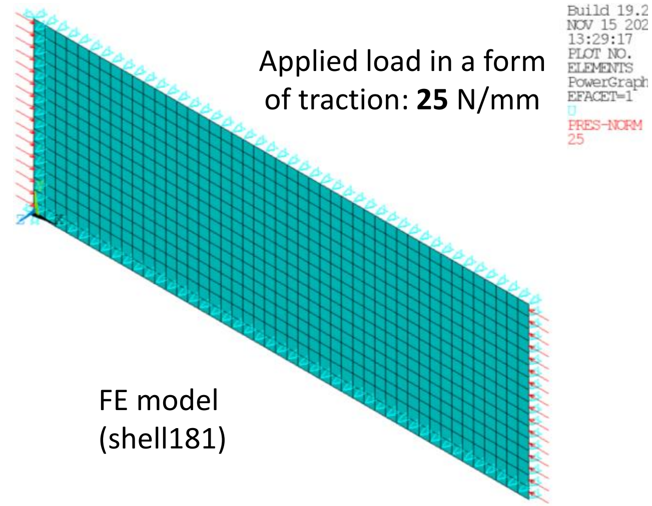
$k = 4$ → $\sigma_{cr} = 17.7\ MPa$





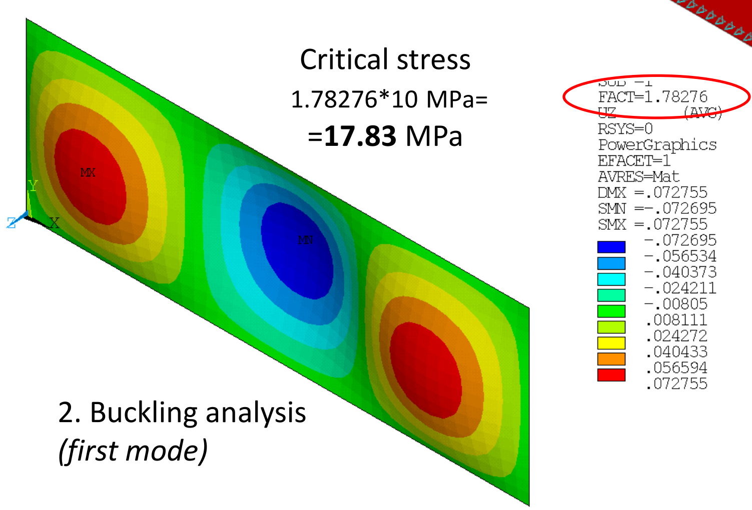
Compressive stress **10** MPa

1. Static analysis (PRESTRESS ON)



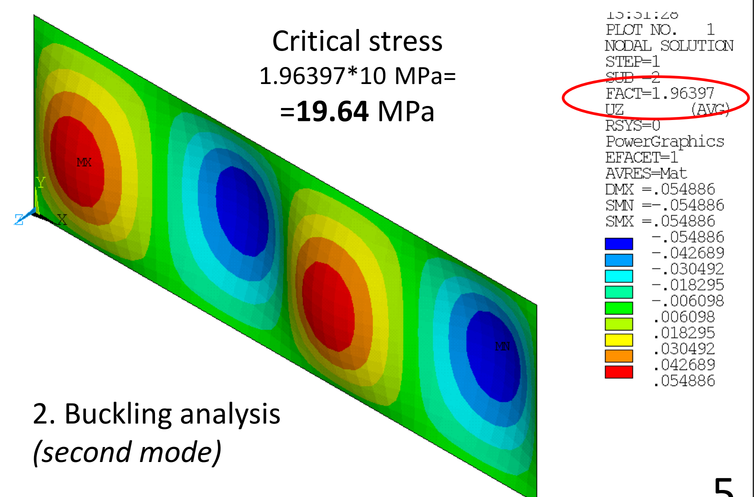
Applied load in a form of traction: **25** N/mm

FE model (shell181)



Critical stress
1.78276*10 MPa=
=**17.83** MPa

2. Buckling analysis *(first mode)*

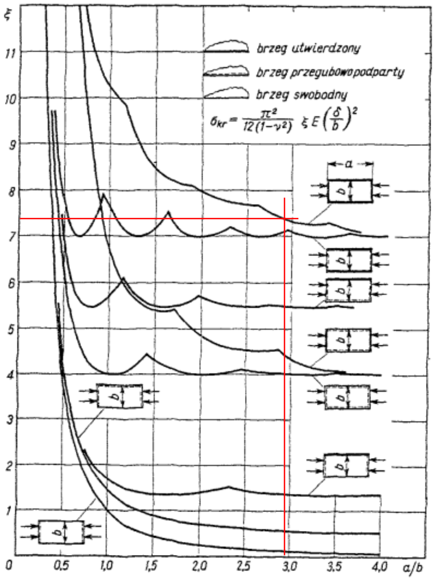


Critical stress
1.96397*10 MPa=
=**19.64** MPa

2. Buckling analysis *(second mode)*

Example 3. Compression of a plate hinged at the edges (uz=0)

a= 885mm, b= 302mm, h = 2.5mm, E=70000 MPa, ν=0.33

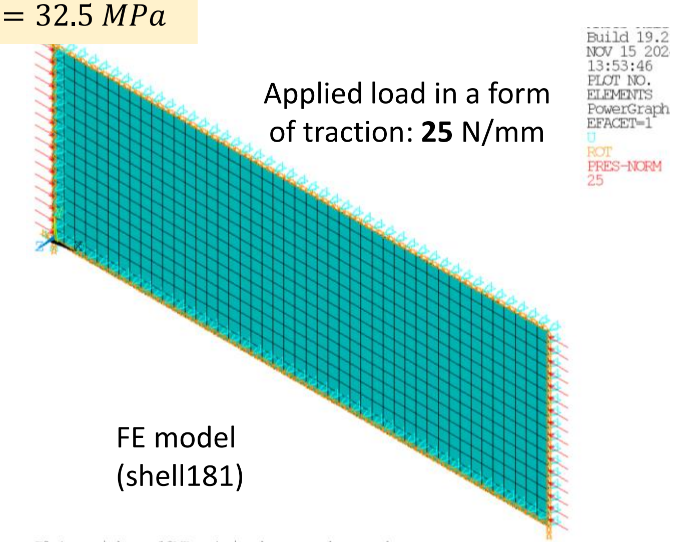


$$\sigma_{cr} = \xi \frac{\pi^2}{12(1-\nu^2)} \frac{Eh^2}{b^2}$$

(theoretical solution)

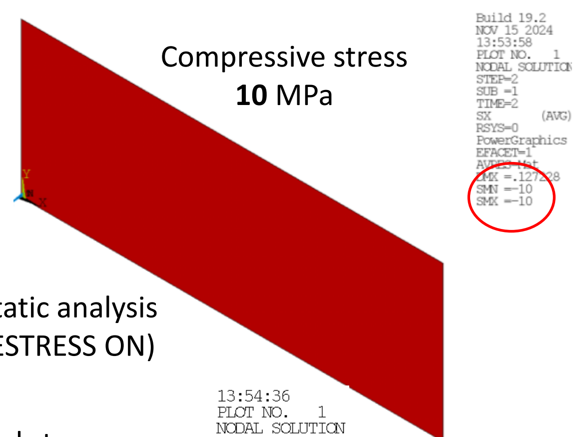
$\xi = 7.35$ → $\sigma_{cr} = 32.5\ MPa$

Applied load in a form of traction: **25** N/mm



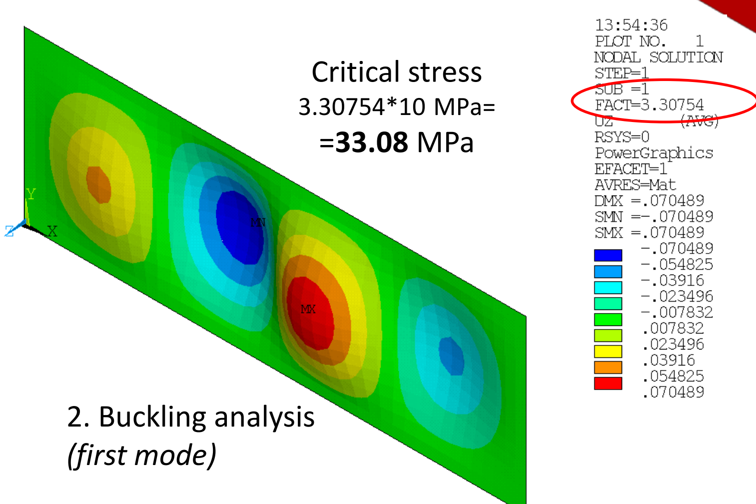
FE model (shell181)

Compressive stress **10** MPa



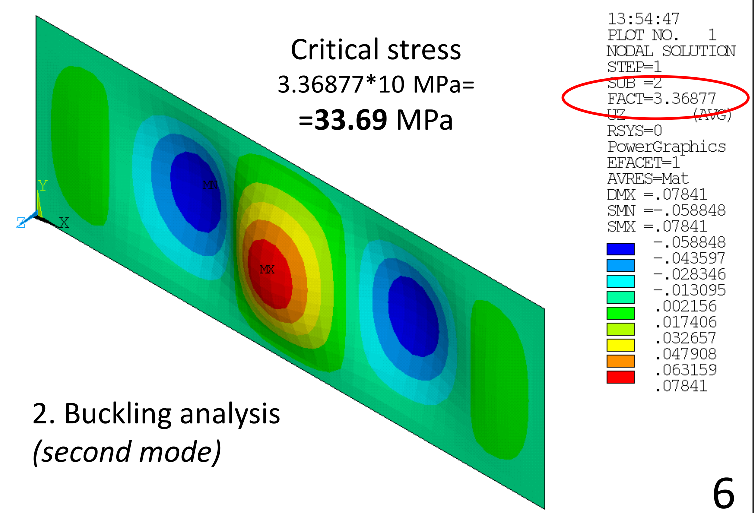
1. Static analysis (PRESTRESS ON)

Critical stress 3.30754*10 MPa= =**33.08** MPa



2. Buckling analysis *(first mode)*

Critical stress 3.36877*10 MPa= =**33.69** MPa



2. Buckling analysis *(second mode)*

Example 4. Shear load in a plate with stringers

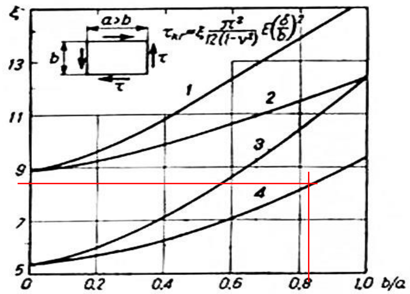
Plate: a=630mm, b=520mm, h=2mm, E=45926 MPa, ν=0.33.

Frame: A_p=1000 mm^2 ($E_p=2\cdot10^5$ MPa, ν_p=0,33)

The structure is loaded by the force F=1000 N.

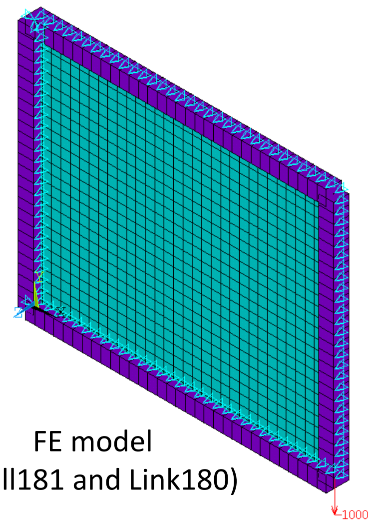
$$\tau_{cr} = \xi \frac{\pi^2}{12(1-\nu^2)} \frac{Eh^2}{b^2}$$

(theoretical solution)



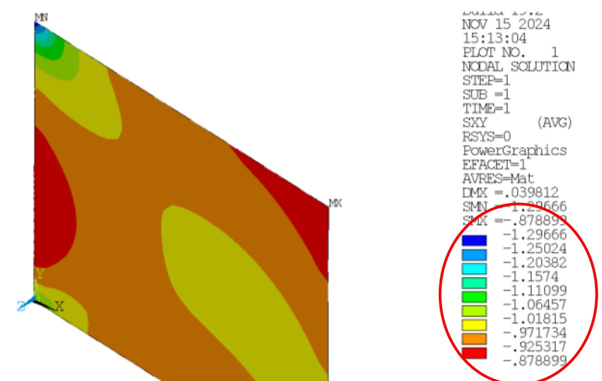
$$\boldsymbol{\tau_{cr}} = 8.25 \frac{\pi^2}{12(1-0.33^2)} \frac{45926\cdot 2^2}{520^2} = \mathbf{5.18}\,MPa$$

F$_{cr}$=5.18MPa*520mm*2mm=**5387**N



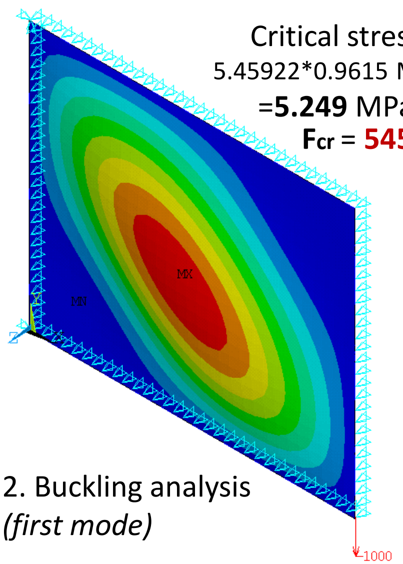
FE model
(shell181 and Link180)

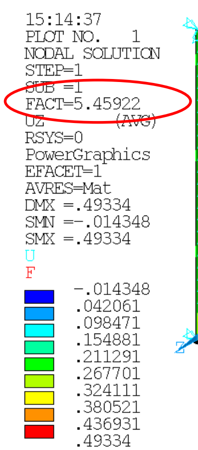
1. Static analysis
(PRESTRESS ON)



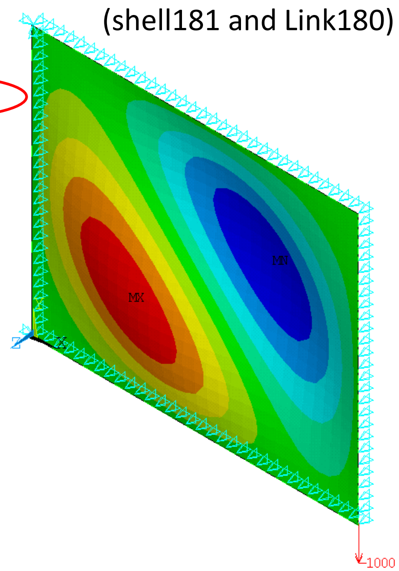
Average shear stress:
1000N/520mm/2mm=
=**0.9615** MPa

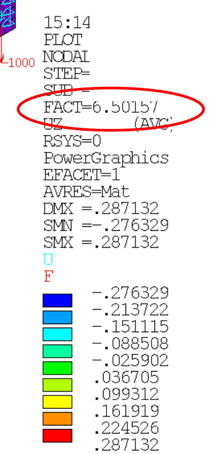
Critical stress
5.45922*0.9615 MPa=
=**5.249** MPa
F$_{cr}$ = **5459** N





2. Buckling analysis
(first mode)

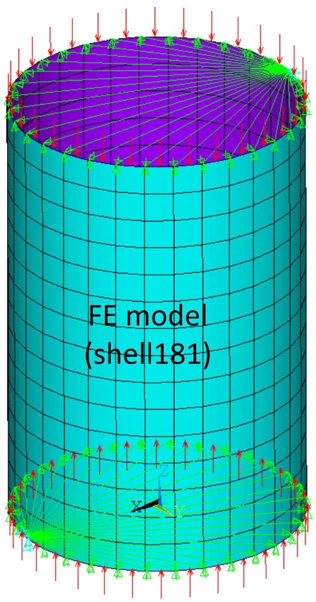


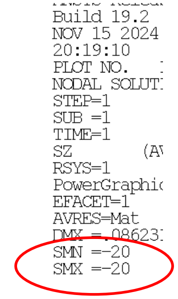


2. Buckling analysis
(second mode)

Example 5. A cylindrical shell: R=100mm, H=300mm, h=0.5mm, E=7e4 MPa, ν=0.33

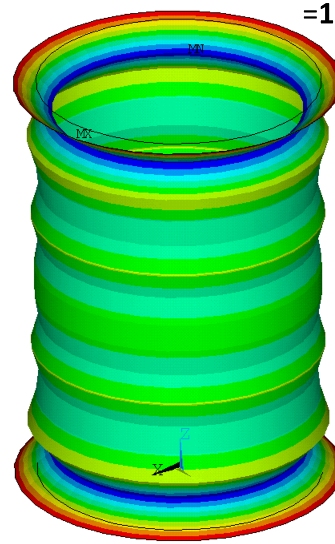
Compressive load

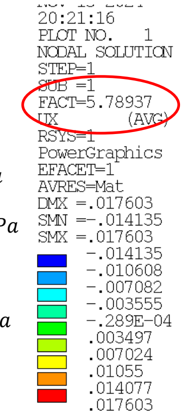




1. Static analysis
(PRESTRESS ON)

Critical load 5.78937*20 MPa=
=**115.8** MPa





$\sigma_{kr}^{A} \approx 210MPa$

$\sigma_{kr}^{B} \approx 129.5MPa$

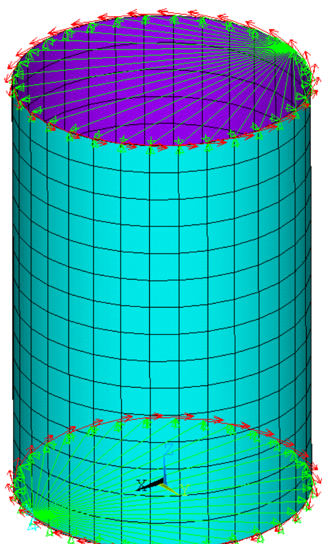
$\sigma_{kr}^{D} \approx 91MPa$

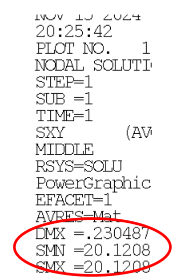
$\sigma_{kr}^{D1} \approx 66.5MPa$

2. Buckling analysis
(first mode)

walec sciskany 20MPa

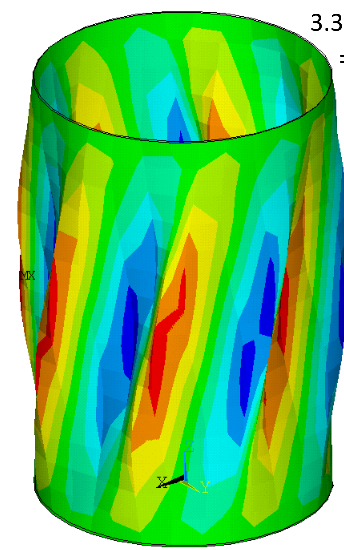
Torsion load

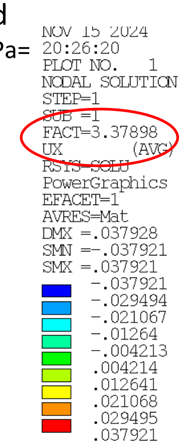




1. Static analysis
(PRESTRESS ON)

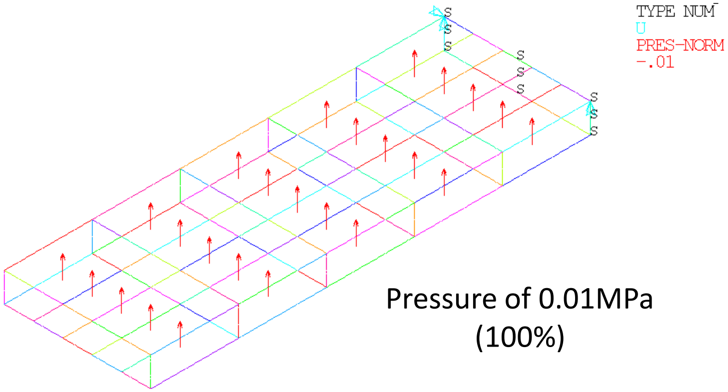
Critical load
3.37898*20 MPa=
=**67.6** MPa





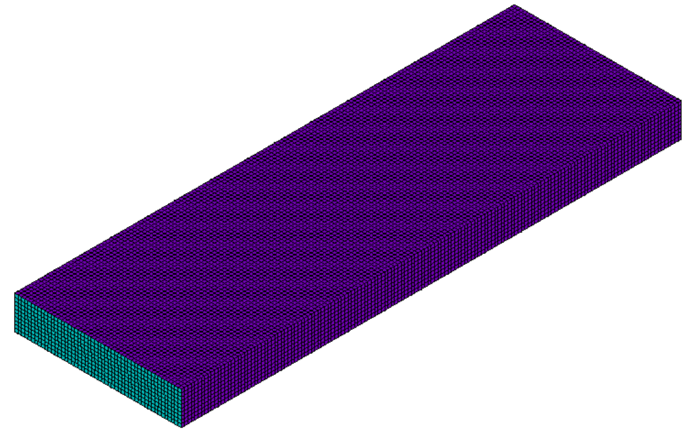
2. Buckling analysis
(first mode)

Example 6a. Bending of a caisson without stringers : L=1500mm, B=300, H=100, G=0.5, E=7e4 MPa, ν=0.33

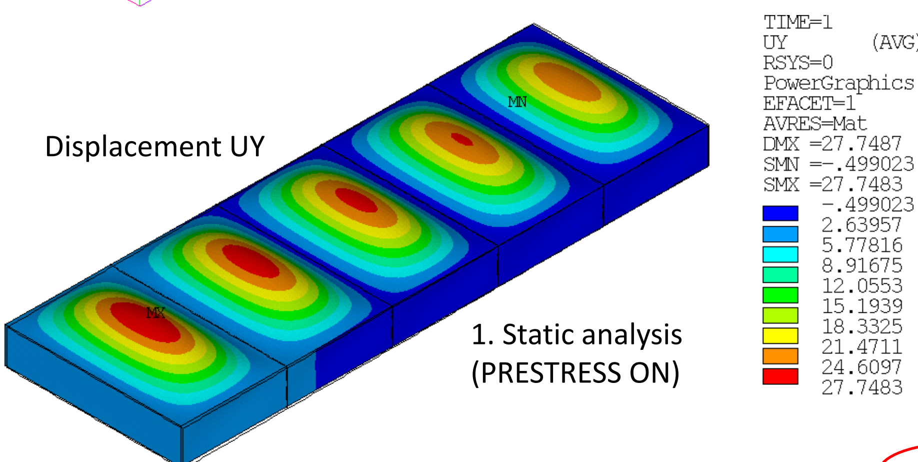


Pressure of 0.01MPa
(100%)

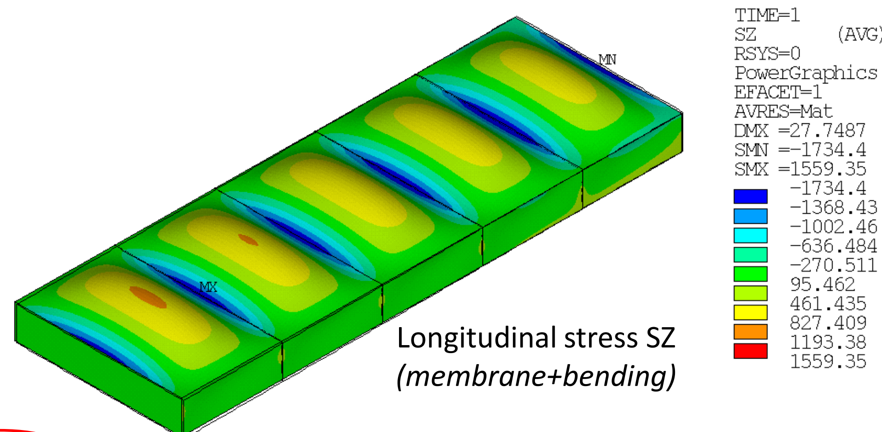
```
*SET,H,100 ! Height
*SET,B,500 ! width
*SET,L,1500 ! length
*SET,n_st,0 ! number of stringers
*SET,th,0.5 ! thickness of cover
*SET,R_st,5 ! radius of stringer
*SET,th_r,2 ! thickness of rib
*SET,E_SIZE,10 ! size of elements
*SET,F_SILA,1000 ! vertical force
*SET,pressure,0.01
*SET,EE,7e5 *SET,NI,.32
```



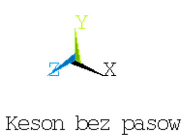
Displacement UY



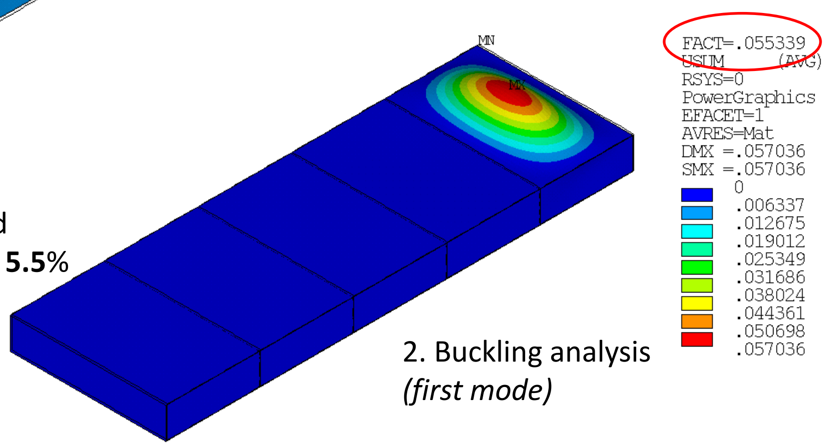
1. Static analysis
(PRESTRESS ON)



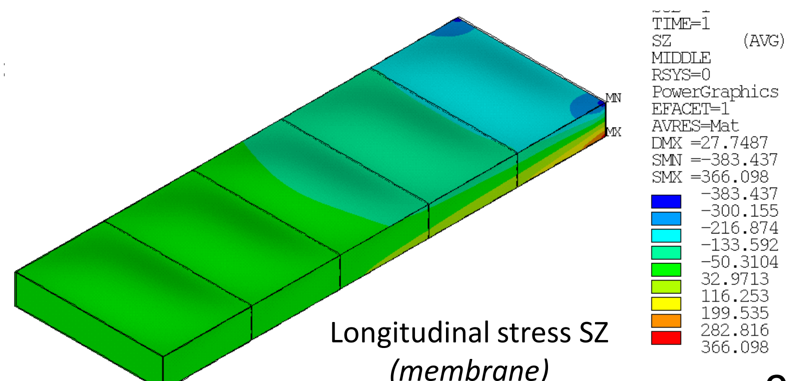
Longitudinal stress SZ
(membrane+bending)



Critical load
.05534*.01 MPa→ **5.5%**

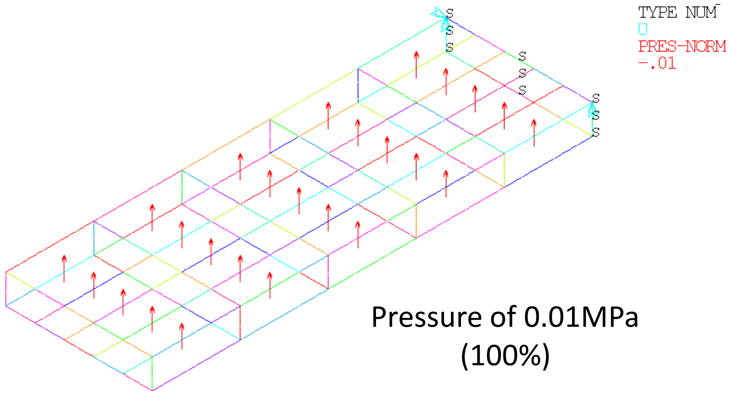


2. Buckling analysis
(first mode)



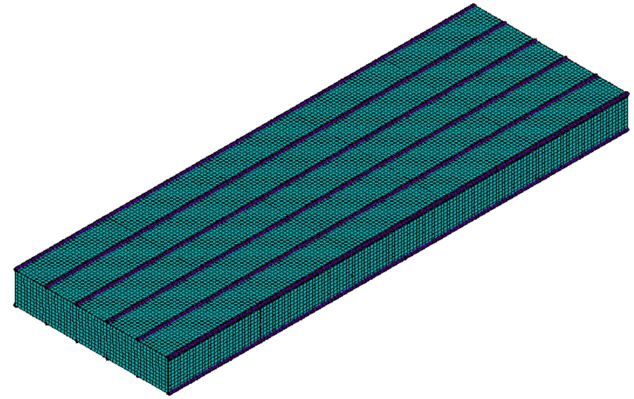
Longitudinal stress SZ
(membrane)

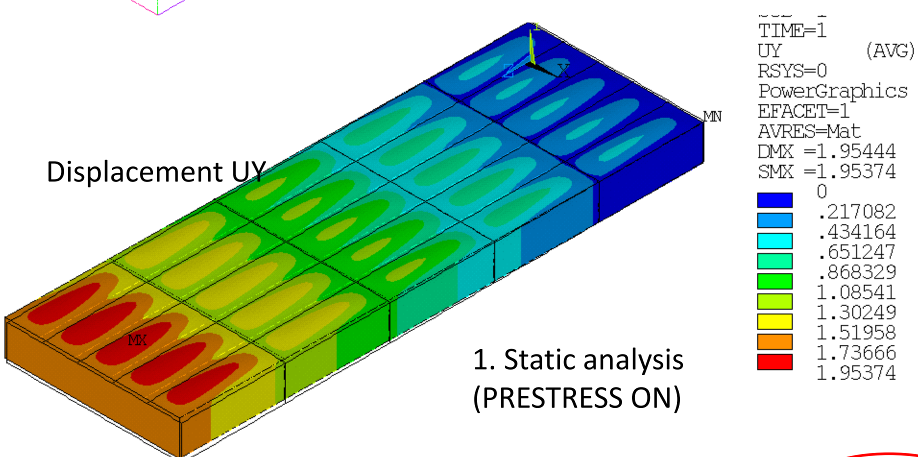
Example 6b. Bending of a caisson with stringers: L=1500mm, B=300, H=100, G=0.5, E=7e4 MPa, ν=0.33



Pressure of 0.01MPa
(100%)

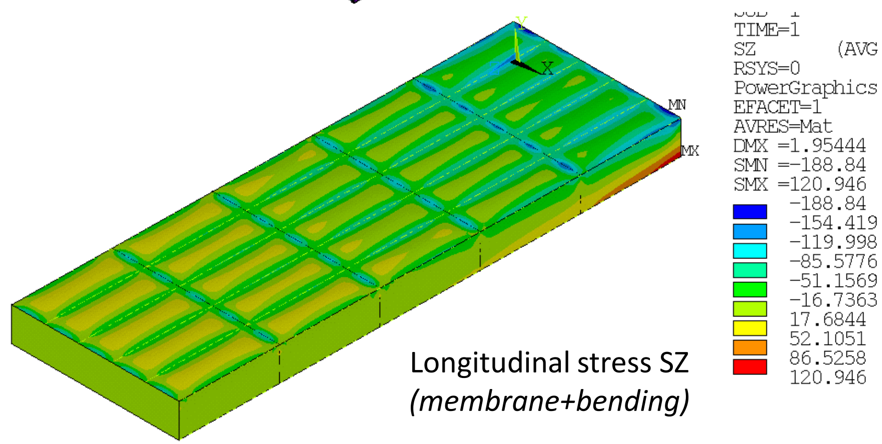
```
*SET,H,100 ! Height
*SET,B,500 ! width
*SET,L,1500 ! length
*SET,n_st,12 ! number of stringers
*SET,th,0.5 ! thickness of cover
*SET,R_st,5 ! radius of stringer
*SET,th_r,2 ! thickness of rib
*SET,E_SIZE,10 ! size of elements
*SET,F_SILA,1000 ! vertical force
*SET,pressure,0.01
*SET,EE,7e5 *SET,NI,.32
```



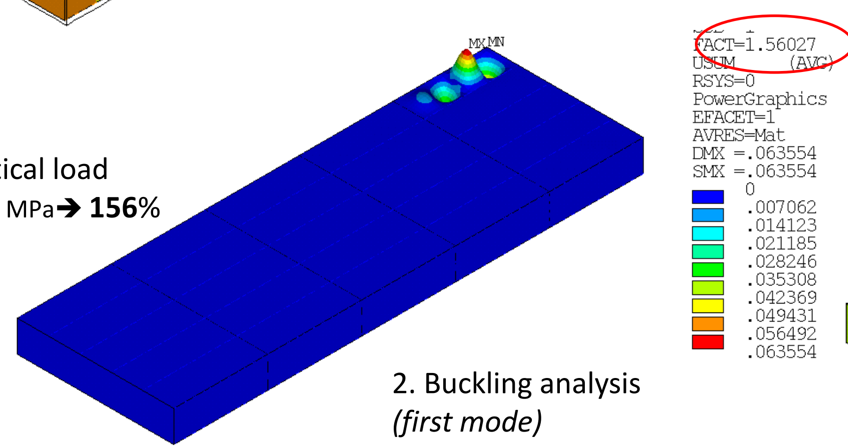


Displacement UY

1. Static analysis
(PRESTRESS ON)

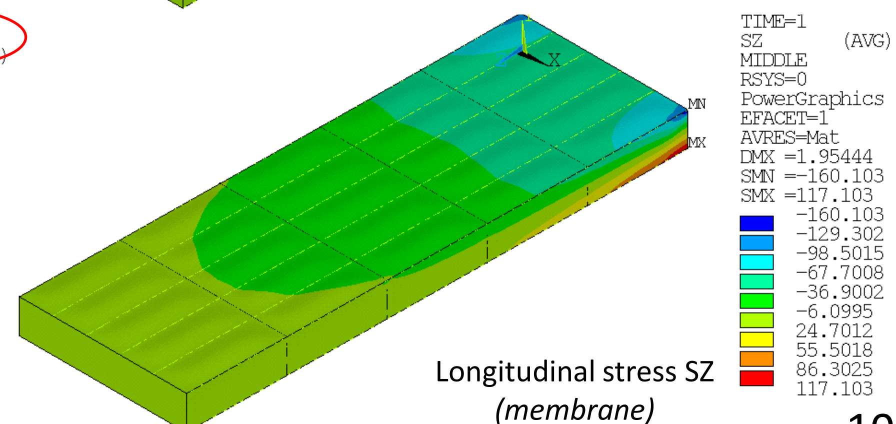


Longitudinal stress SZ
(membrane+bending)



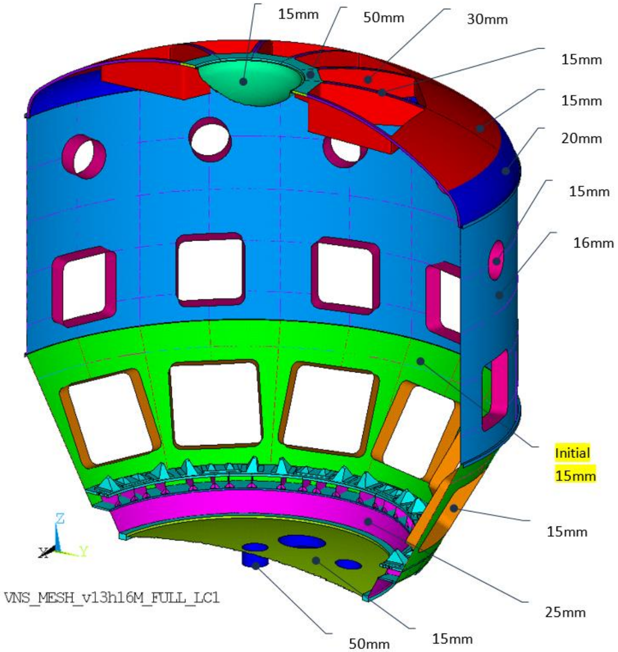
Critical load
1.56*.01 MPa→ **156%**

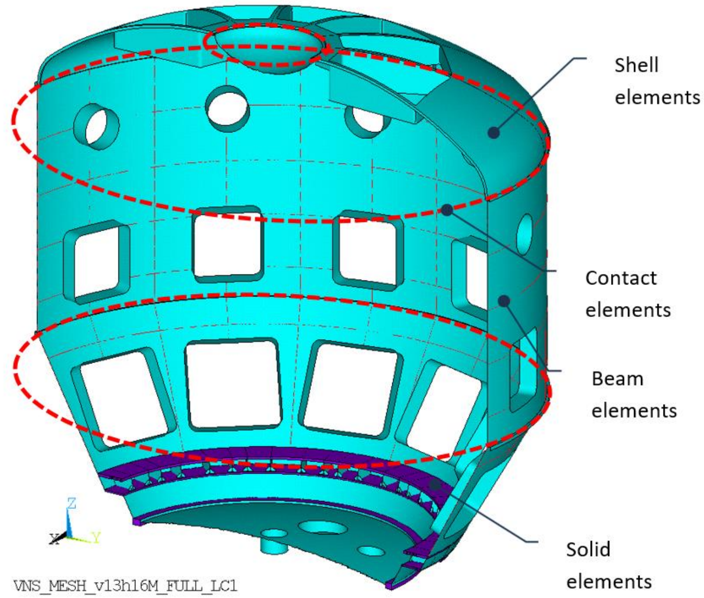
2. Buckling analysis
(first mode)

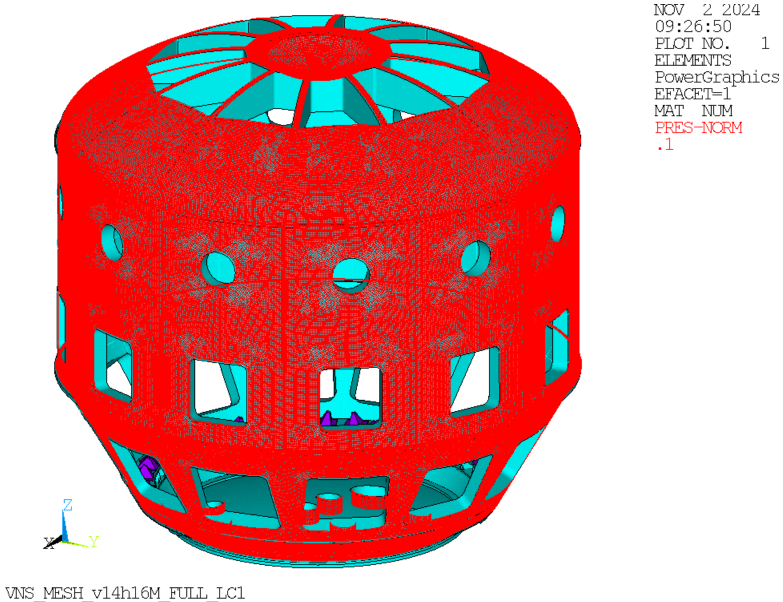


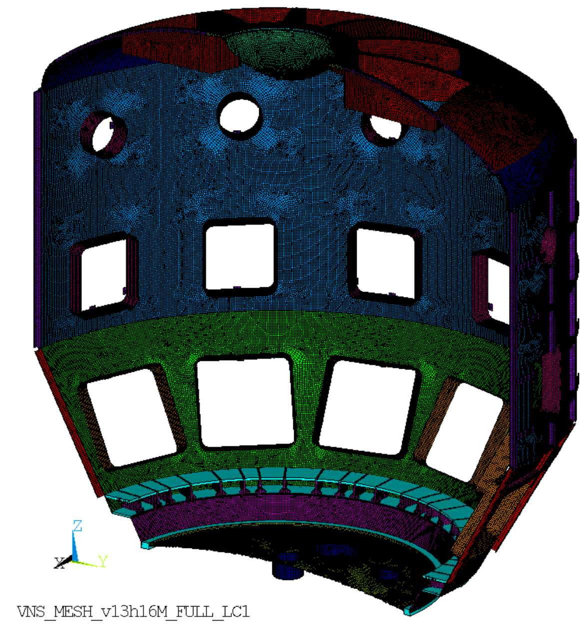
Longitudinal stress SZ
(membrane)

Example 7. FEM analysis of a cryostat (VNS Feasibility study 2024)

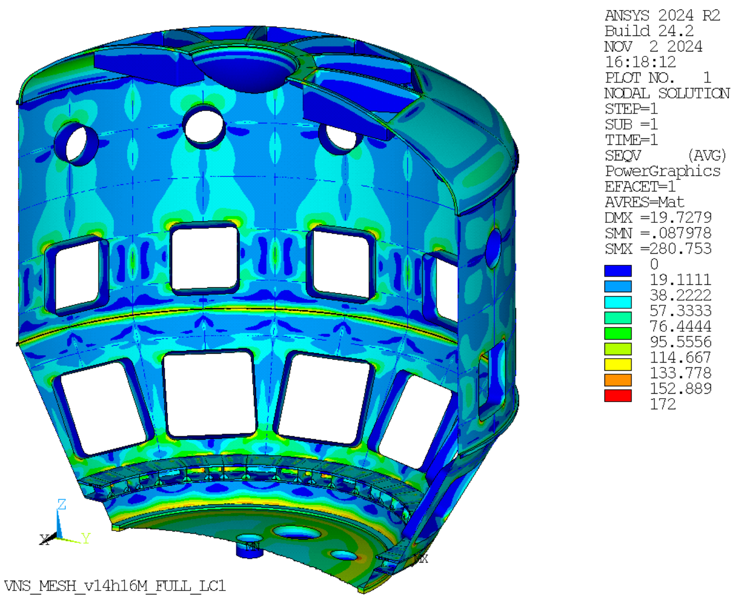






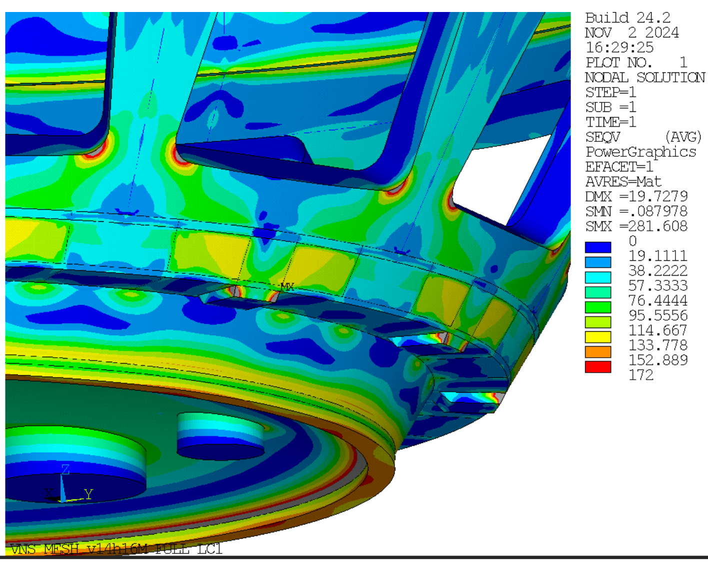


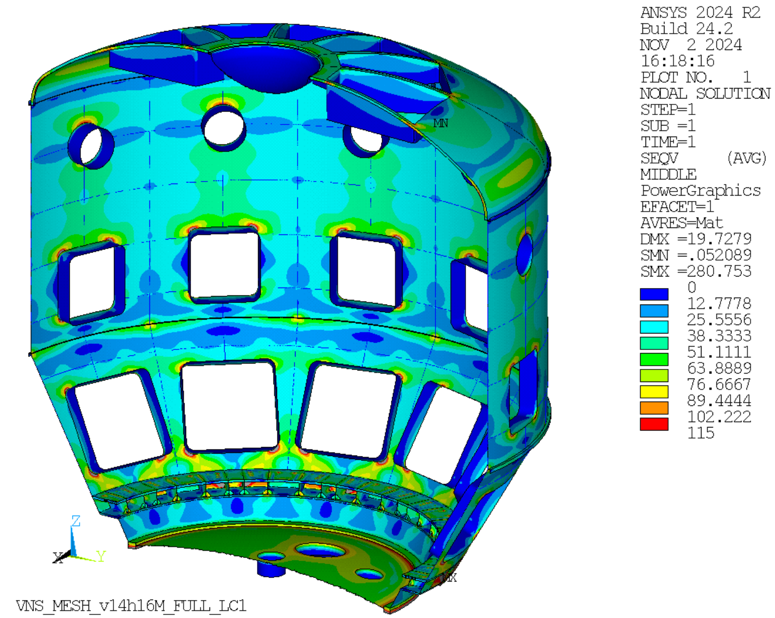
Load case 1: **Normal operation** (P + D)



Von Mises stress (Pm+Pb) [MPa]

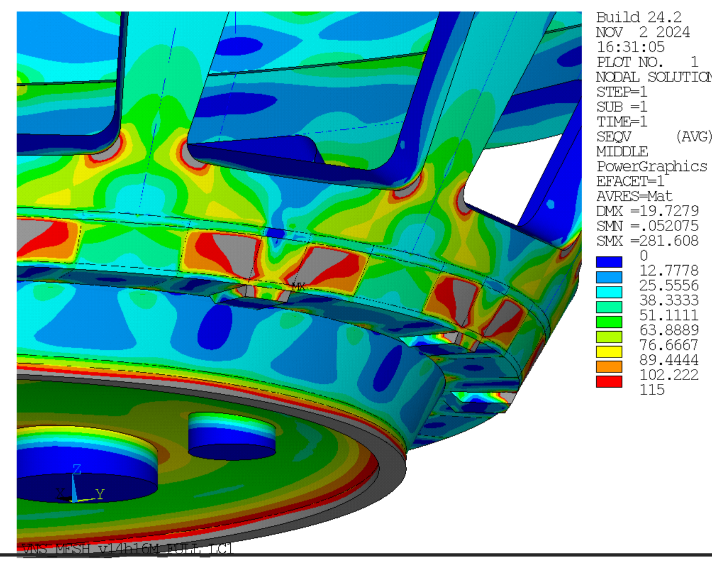
Load case 1: **Normal operation** (P + D)



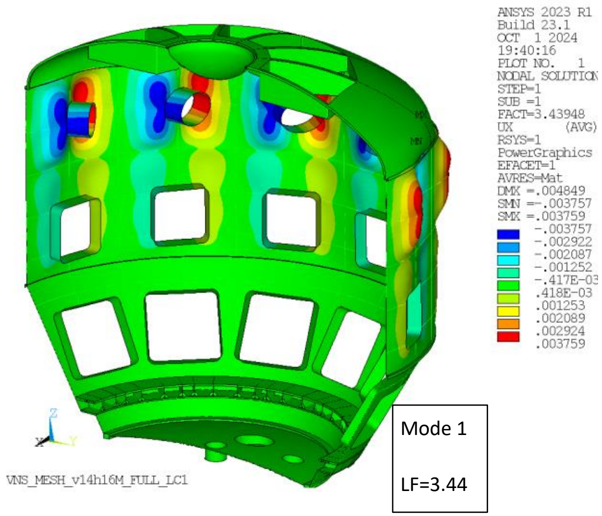


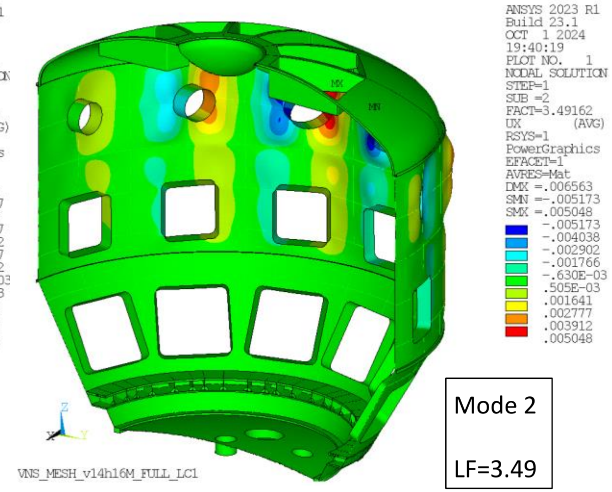
Von Mises membrane stress (Pm) [MPa]

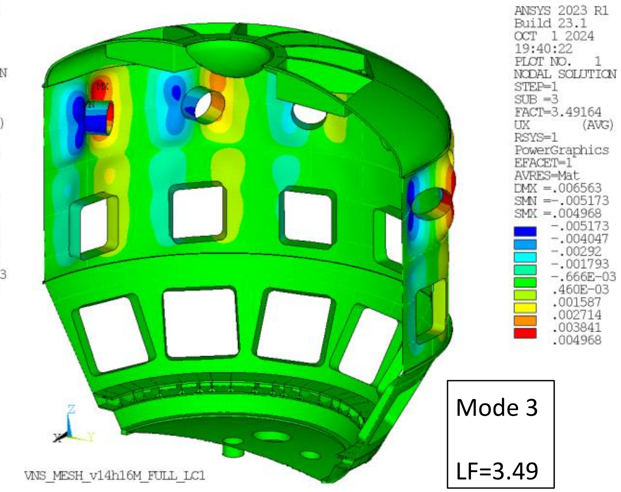
Load case 1: **Normal operation** (P + D)

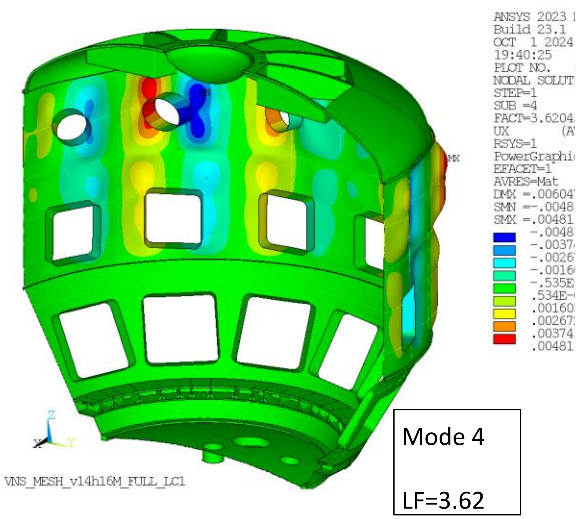


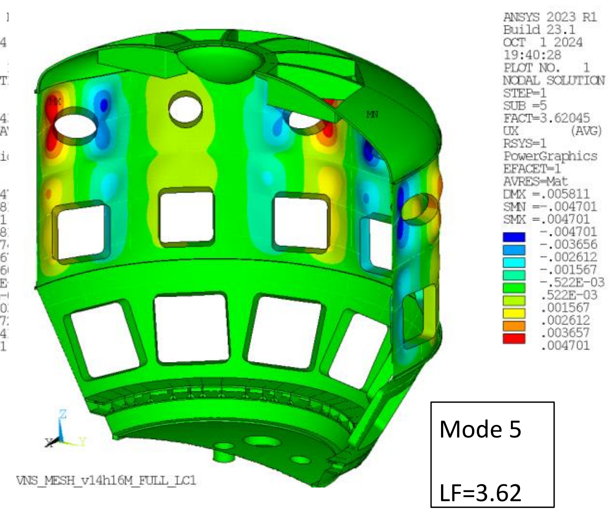
Buckling modes 1-6 for **Load case 1**: **Normal operation** (P + D)

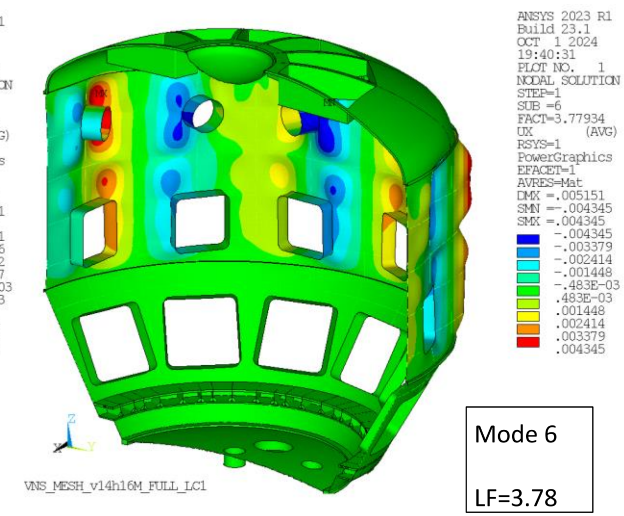












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Protection against Collapse from Buckling - Elastic – Plastic Analysis 2.4 (P + D)

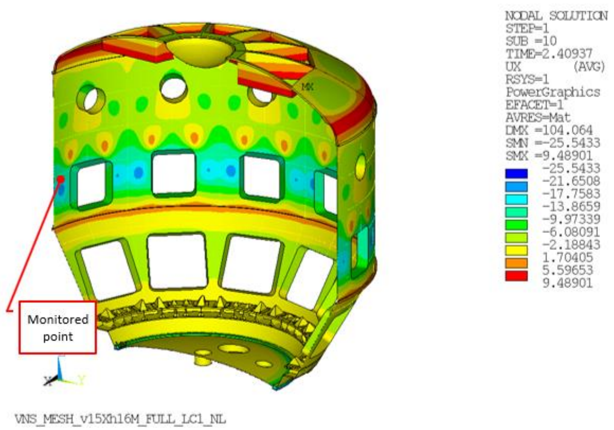
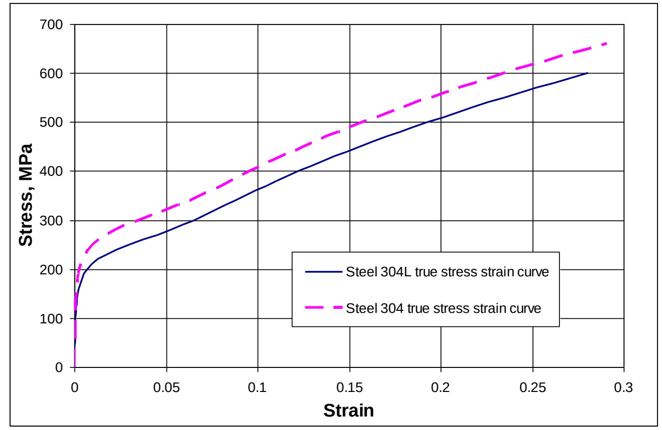


Figure 5-9 Radial displacements [mm] for **LF=2.41**- Final model: **Elastic – Plastic Analysis 2.4 (P + D)**



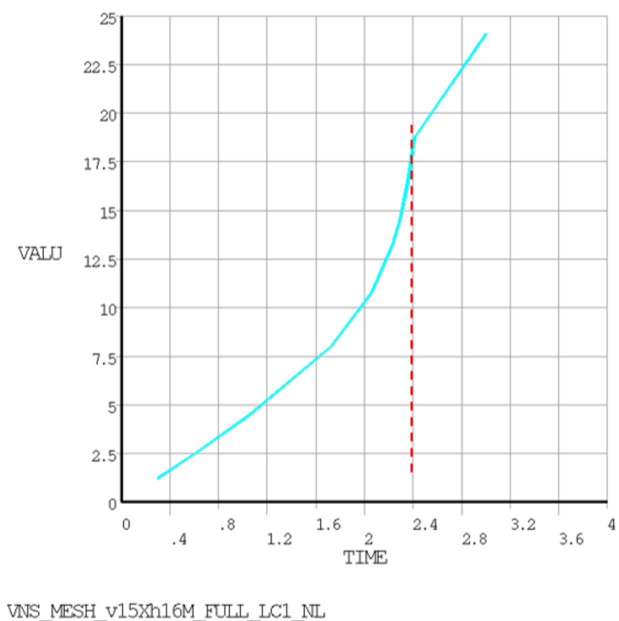
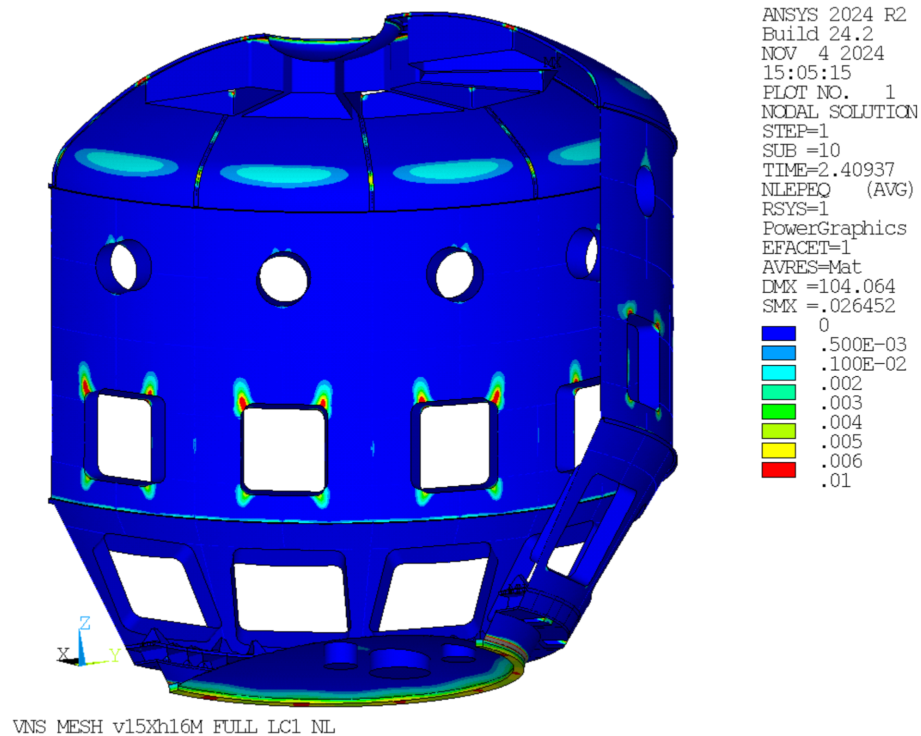


Figure 5-11 Radial displacement of the monitored point as a functions of the Load Factor - Final model - **Elastic – Plastic Analysis 2.4 (P + D)**



Accumulated Equivalent plastic strain for **LF=2.41**- Final model:
Elastic – Plastic Analysis 2.4 (P + D)