Computer Science II – Lab 4

The objective is to solve numerically the following initial value problem (IVP)

$$\begin{cases} y'(t) = -y(t) + e^{-t}\cos(t) \quad , \quad t > 0 \\ y(0) = 0 \end{cases}$$

using the Euler and 4[th]-order Runge-Kutta methods. The exact solution is $Y(t) = e^{-t}\sin(t)$.

1. Write the C function ***double derivative(double t, double y)*** which calculates the right-hand side of the differential equation, i.e. $f(t, y) = -y + e^{-t}\cos(t)$.

2. Write the C function ***double solution(double t, double y)*** which calculates Y(t).

3. Write the main function which solves the IVP using the Euler method. The following parameters are to be read from the keyboard: H – the time step, NS – the number of time steps. This way, the integration interval is [0,NS*H]. The formula of the Euler's method is

   $$y_{k+1} = y_k + H \cdot f(t_k, y_k) \quad , \quad k = 0,1,..,NS-1$$

   where, in our case, $y_0 = 0$. At each step, dump the values of t_k, y_k , $Y(t_k)$ and $e_k = Y(t_k) - y_k$ to an output file on the hard disk.

4. Repeat calculations with (H, NS) = (0.01, 3000), (0.005, 6000) and (0.0025, 12000). Compare the error distributions on the same plot using Grapher.

5. Write a new version of the main function which uses the 4[th] order R-K method, i.e.

   $$\begin{aligned} q_1 &= H \cdot f(t_k, y_k) \\ q_2 &= H \cdot f(t_k + H/2, y_k + q_1/2) \\ q_3 &= H \cdot f(t_k + H/2, y_k + q_2/2) \\ q_4 &= H \cdot f(t_k + H, y_k + q_3) \\ y_{k+1} &= y_k + (q_1 + 2q_2 + 2q_3 + q_4)/6 \end{aligned}$$

6. Repeat the calculations with three different step sizes H and the corresponding numbers of steps NS. Compare the numerical accuracy of the 4[th] order R-K and the Euler methods.