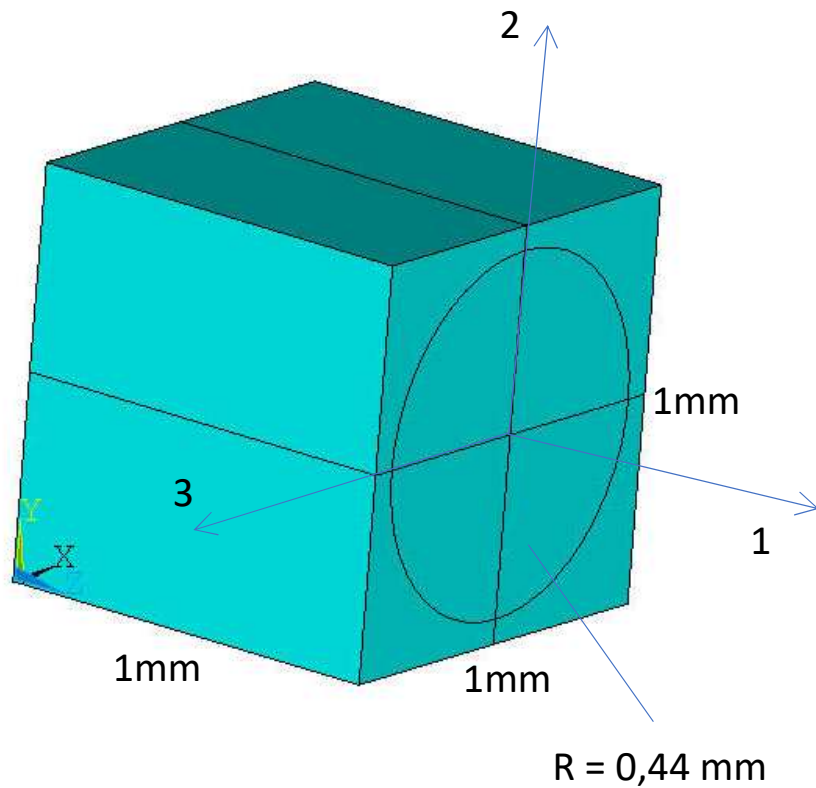


Homogenizacja numeryczna



Włókno szklane:

$$E_w = 73500 \text{ MPa}$$

$$\nu_w = 0,22$$

Spoivo epoksydowe:

$$E_s = 3400 \text{ MPa}$$

$$\nu_s = 0,35$$

$$V_w = 0,44^2 \pi / 1 \approx 0,61$$

$$V_s \approx 0,39$$

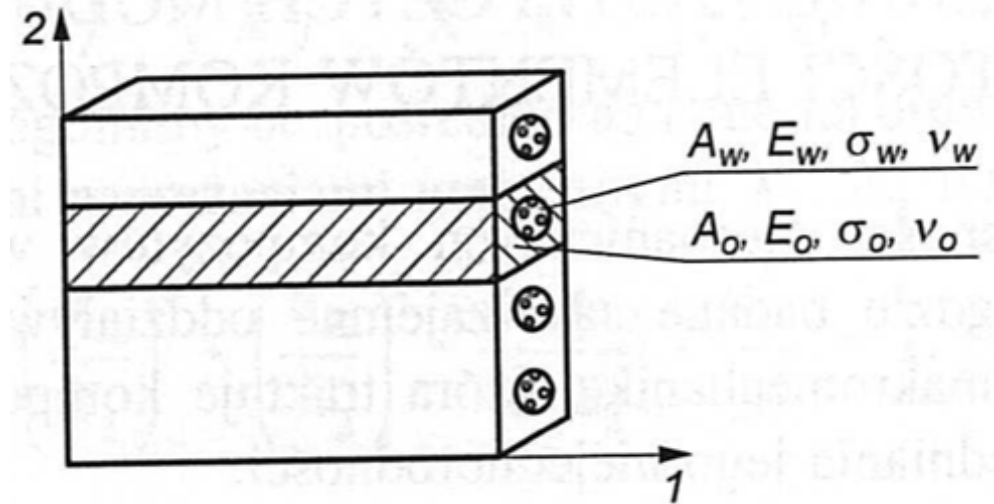
$$G = \frac{E}{2(1 + \nu)} \rightarrow G_w = \frac{73500}{2 \cdot 1,22} = 30123 \text{ MPa}; G_s = \frac{3400}{2 \cdot 1,35} = 1259 \text{ MPa}$$

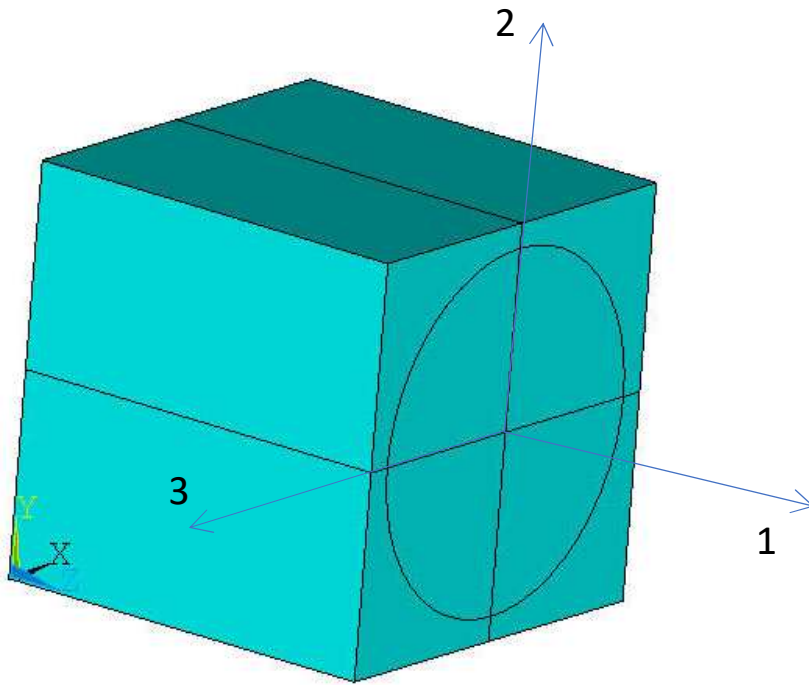
$$E_1 = E_w V_w + E_s V_s = 46161 \text{ MPa}$$

$$E_2 = \frac{E_w \cdot E_s}{E_s V_w + E_w V_s} = 8130 \text{ MPa}$$

$$\nu_{12} = \nu_w V_w + \nu_s V_s = 0,27$$

$$G_{12} = \frac{G_w \cdot G_s}{G_s V_w + G_w V_s} = 2010 \text{ MPa}$$





$$E_2 = E_3$$

$$\nu_{12} = \nu_{13}$$

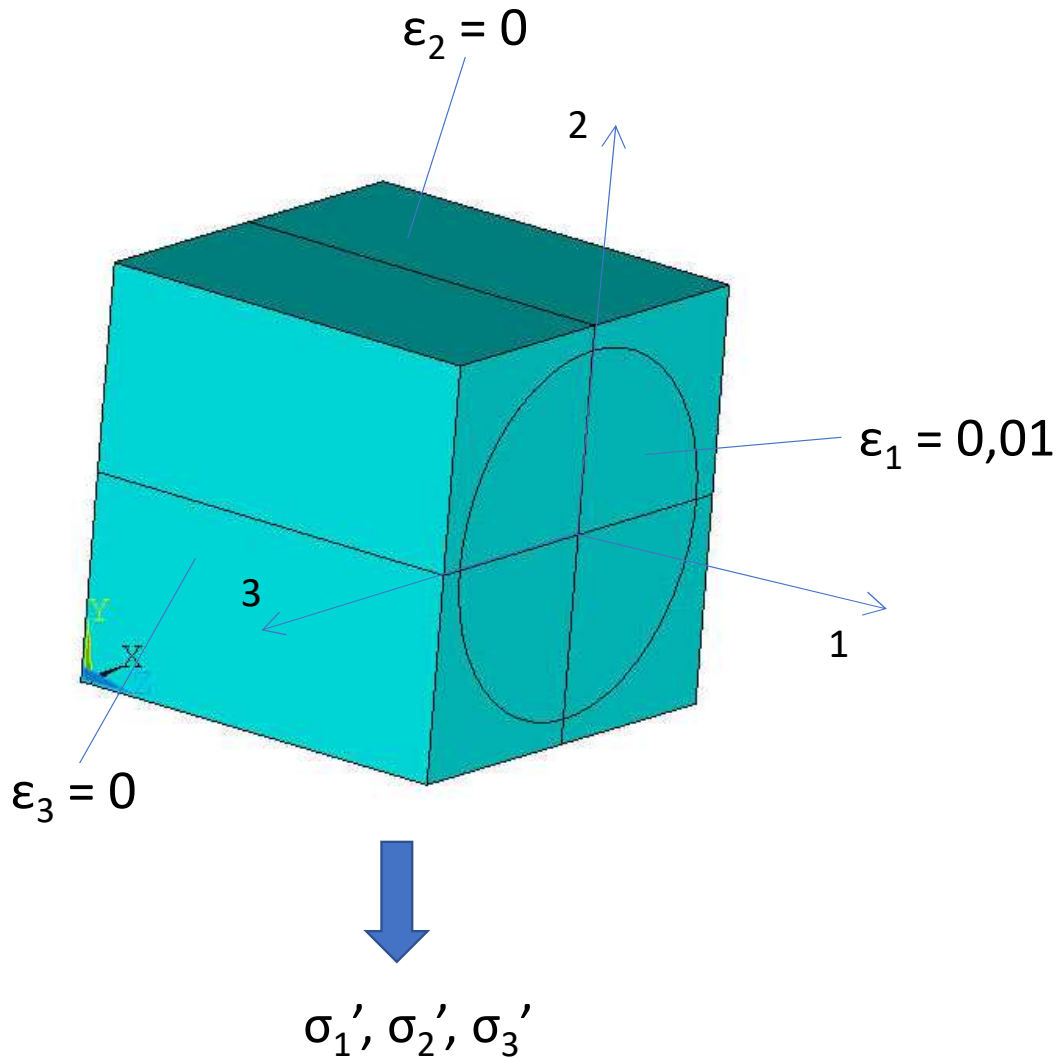
$$G_{12} = G_{13}$$

Zależność między odkształceniem i naprężeniem

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} \quad \rightarrow \quad \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}$$

$E_1, E_2, E_3, \nu_{12}, \nu_{13}, \nu_{23}$

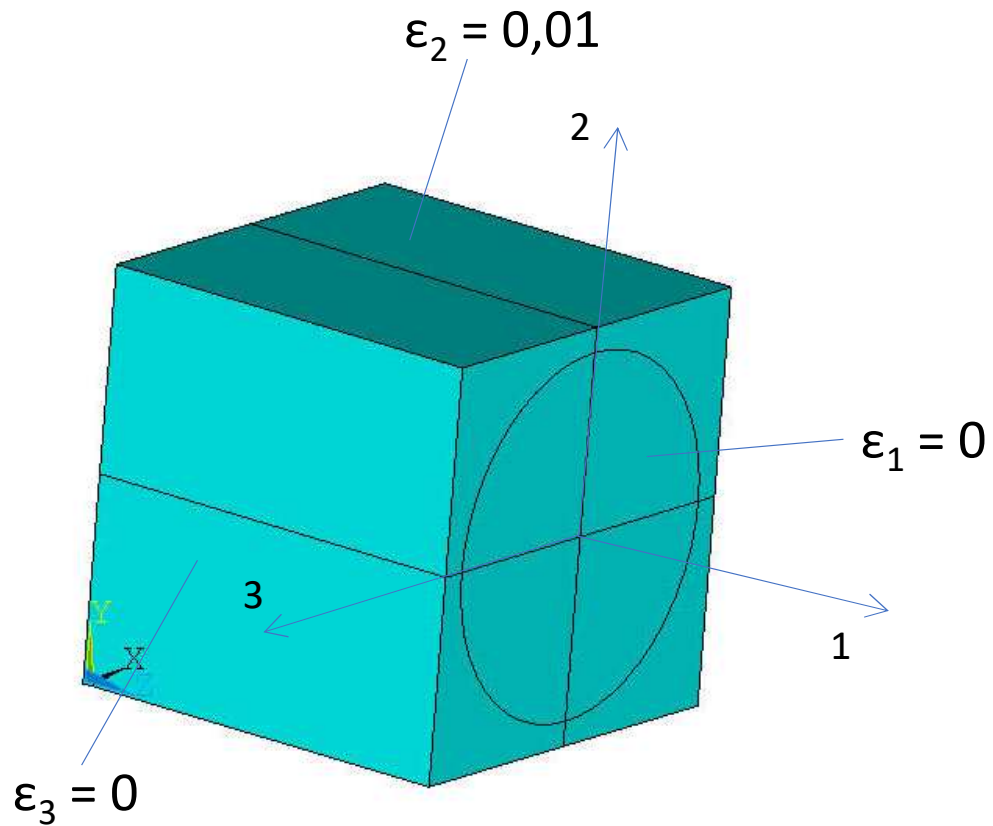


$$\begin{Bmatrix} 0,01 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} \end{bmatrix} \begin{Bmatrix} \sigma'_1 \\ \sigma'_2 \\ \sigma'_3 \end{Bmatrix}$$

$$0,01 = \frac{1}{E_1} \sigma'_1 - \frac{\nu_{12}}{E_1} (\sigma'_2 + \sigma'_3)$$

$$0 = -\frac{\nu_{12}}{E_1} \sigma'_1 + \frac{1}{E_2} \sigma'_2 - \frac{\nu_{23}}{E_2} \sigma'_3$$

$$0 = -\frac{\nu_{12}}{E_1} \sigma'_1 - \frac{\nu_{23}}{E_2} \sigma'_2 + \frac{1}{E_2} \sigma'_3$$



↓

$\sigma_1'', \sigma_2'', \sigma_3''$

$$\begin{Bmatrix} 0 \\ 0,01 \\ 0 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} \end{bmatrix} \begin{Bmatrix} \sigma_1'' \\ \sigma_2'' \\ \sigma_3'' \end{Bmatrix}$$

$$0 = \frac{1}{E_1} \sigma_1'' - \frac{\nu_{12}}{E_1} (\sigma_2'' + \sigma_3'')$$

$$0,01 = -\frac{\nu_{12}}{E_1} \sigma_1'' + \frac{1}{E_2} \sigma_2'' - \frac{\nu_{23}}{E_2} \sigma_3''$$

$$0 = -\frac{\nu_{12}}{E_1} \sigma_1'' - \frac{\nu_{23}}{E_2} \sigma_2'' + \frac{1}{E_2} \sigma_3''$$

$$\sigma'_1 = 449,19 \text{ MPa}$$

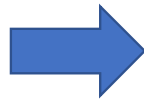
$$\sigma'_2 = 59,37 \text{ MPa}$$

$$\sigma'_3 = 59,34 \text{ MPa}$$

$$\sigma''_1 = 61,62 \text{ MPa}$$

$$\sigma''_2 = 176,69 \text{ MPa}$$

$$\sigma''_3 = 50,82 \text{ MPa}$$



$$0,01 = \frac{1}{E_1} \sigma'_1 - \frac{\nu_{12}}{E_1} (\sigma'_2 + \sigma'_3)$$

$$0 = -\frac{\nu_{12}}{E_1} \sigma'_1 + \frac{1}{E_2} \sigma'_2 - \frac{\nu_{23}}{E_2} \sigma'_3$$

$$0 = -\frac{\nu_{12}}{E_1} \sigma'_1 - \frac{\nu_{23}}{E_2} \sigma'_2 + \frac{1}{E_2} \sigma'_3$$

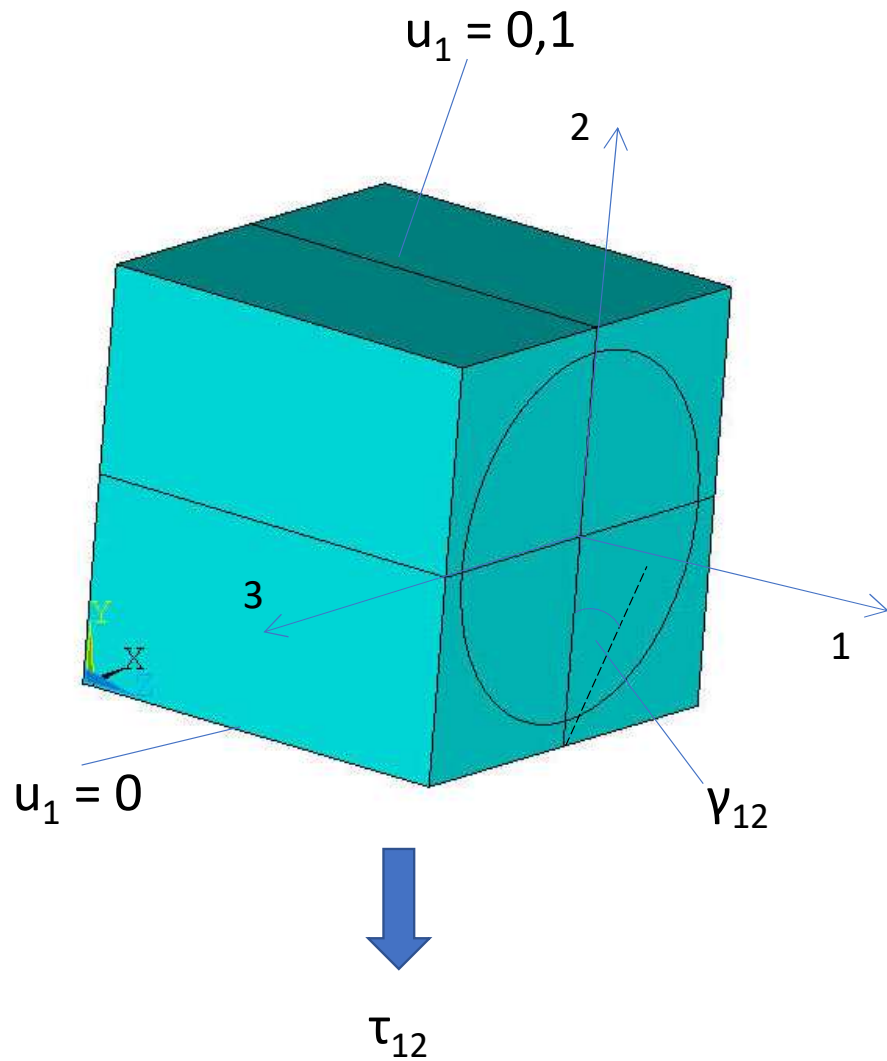
$$0 = \frac{1}{E_1} \sigma''_1 - \frac{\nu_{12}}{E_1} (\sigma''_2 + \sigma''_3)$$

$$0,01 = -\frac{\nu_{12}}{E_1} \sigma''_1 + \frac{1}{E_2} \sigma''_2 - \frac{\nu_{23}}{E_2} \sigma''_3$$

$$0 = -\frac{\nu_{12}}{E_1} \sigma''_1 - \frac{\nu_{23}}{E_2} \sigma''_2 + \frac{1}{E_2} \sigma''_3$$



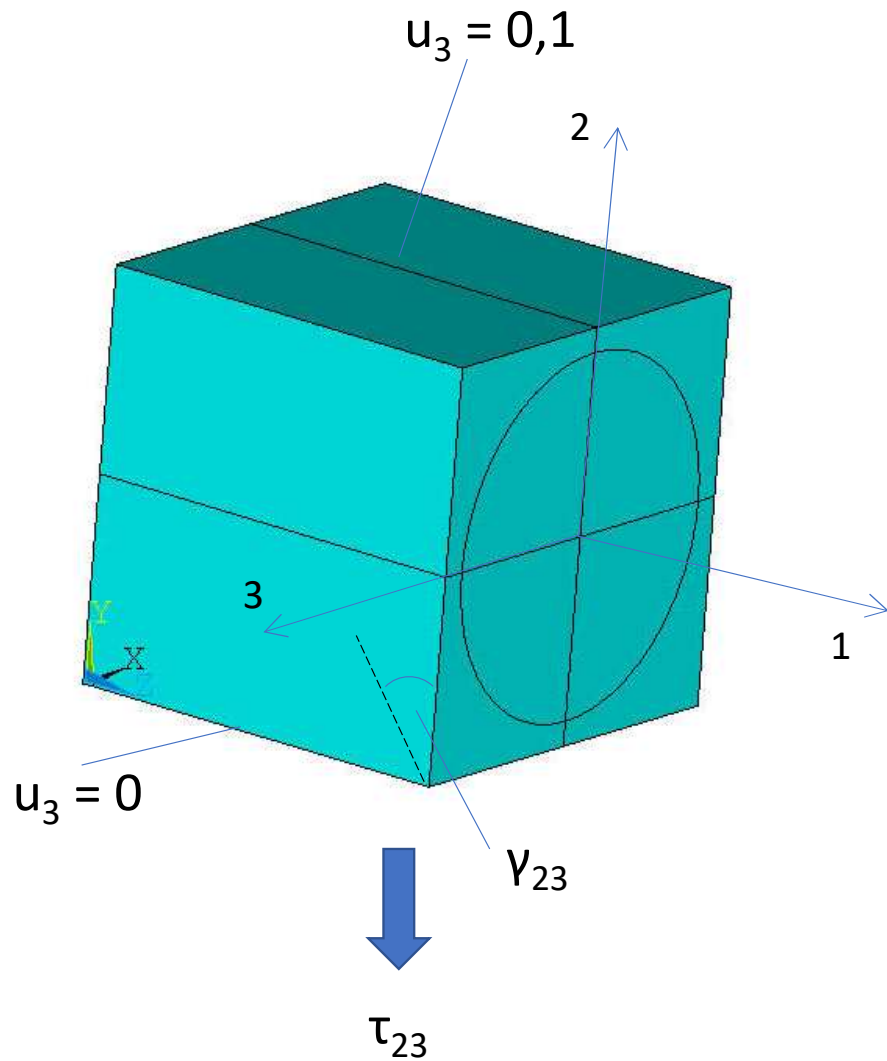
	Homogenizacja analityczna	Homogenizacja numeryczna
E_1	46161 MPa	36307 MPa
E_2	8130 MPa	15294 MPa
E_3	8130 MPa	14810 MPa
ν_{12}	0,27	0,26
ν_{23}	-	0,14
ν_{13}	0,27	0,25



$$\operatorname{tg} \gamma_{12} = \frac{0,1}{1} \rightarrow \gamma_{12} = \operatorname{arctg} 0,1$$

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

$$\tau_{12} = \tau_{13} = 124,19 \text{ MPa}$$



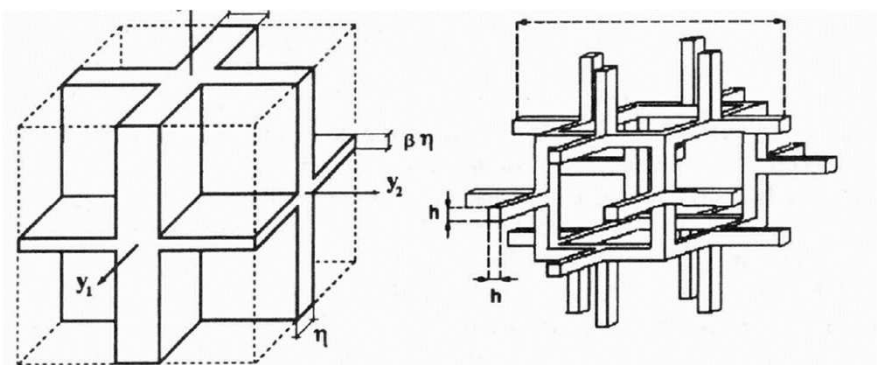
$$\operatorname{tg} \gamma_{23} = \frac{0,1}{1} \rightarrow \gamma_{23} = \operatorname{arctg} 0,1$$

$$G_{23} = \frac{\tau_{23}}{\gamma_{23}}$$

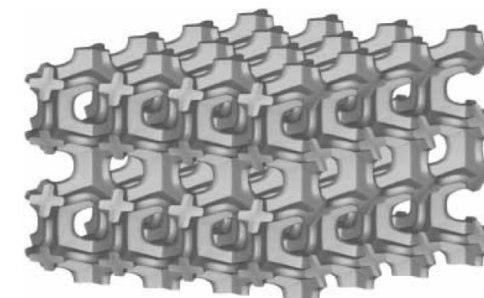
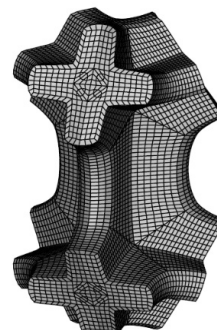
$$\tau_{23} = 134,82 \text{ MPa}$$

	Homogenizacja analityczna	Homogenizacja numeryczna
G_{12}	2010 MPa	1246 MPa
G_{13}	2010 MPa	1246 MPa
G_{23}	–	1352 MPa

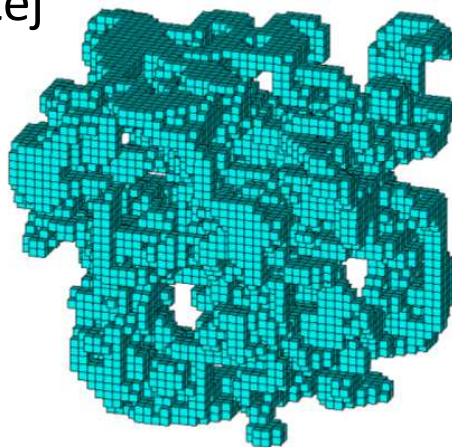
Przykłady zaawansowanej homogenizacji numerycznej



model do analitycznej homogenizacji
periodycznej struktury porowatej



model numeryczny
struktury periodycznej



model numeryczny struktury chaotycznej