Integrated Laboratory

Strength of Materials and Structures

Torsion

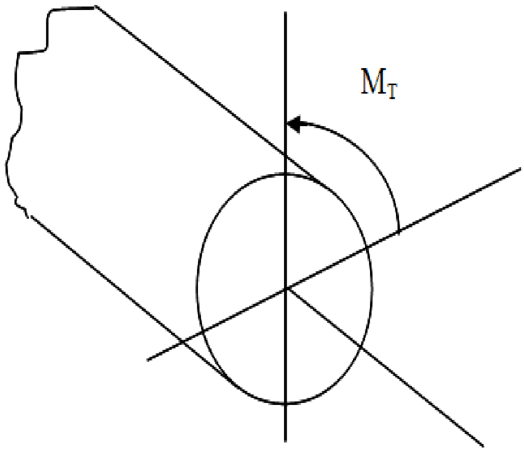
Before attending the laboratory students should recollect the following topics:
torque distribution in loaded rods, stress distribution, twist angles and relative twist angles, plane cross-section hypothesis, Hook's law for shearing, thin walled members: assumptions, Bredt's formulae, shear centre

Recommended Bibliography:

- William A. Nash *Strenght of materials*
- Roy R. Craig *Mechanics of Materials*
- *Mechanika Materiałów i Konstrukcji* edited by Marek Bijak-Żochowski
- Own lecture notes
- the Internet (for lazy students)

1 Basic Formulae

1.1 Circular rods



1.1.1 Deformations

1. Relative twist angle

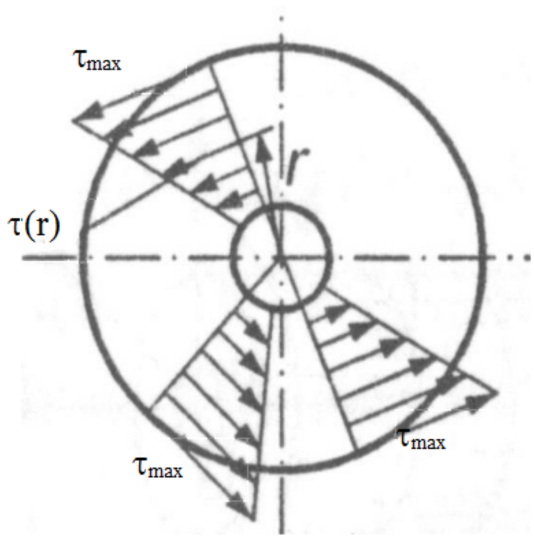
$$\Theta = \frac{M_T}{GJ_0}$$

   where:
   M_T - torque, G - shear modulus, $J_0 = \frac{\pi({r_z}^4 - {r_w}^4}{)}2$ - polar moment of inertia,
   r_z - external radius, r_w - internal radius

2. Twist angle

$$\phi(x) = \frac{M_T}{GJ_0}x$$

1.1.2 Stress Distribution

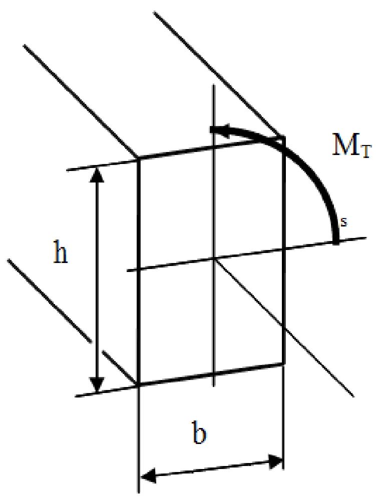


$$\tau(r) = \frac{M_T}{J_0} r$$

where:
M_T - torque, r - radial coordinate, J_0 - polar moment of inertia

$$\tau_{max} = \frac{M_T}{J_0} r_z$$

1.2 Rectangular rods



1.2.1 Deformations

1. Relative Twist Angle

$$\Theta = \frac{M_T}{GJ_S}$$

$$J_S = k_1 b^3 h$$

2. Twist Angle

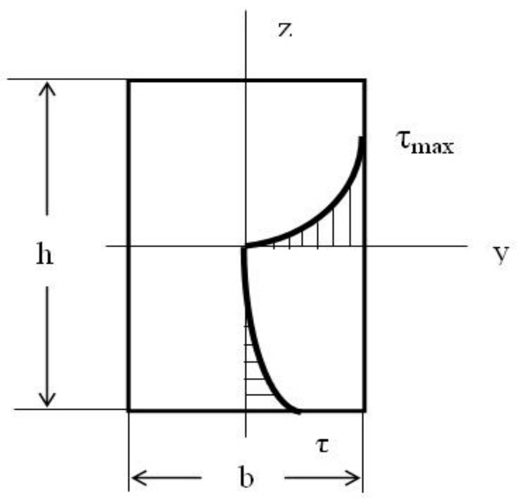
$$\phi = \frac{M_T}{GJ_S} x$$

$$J_S = k_1 b^3 h$$

1.2.2 Stress Distribution

$$\tau_{max} = \frac{M_S}{W_S}$$

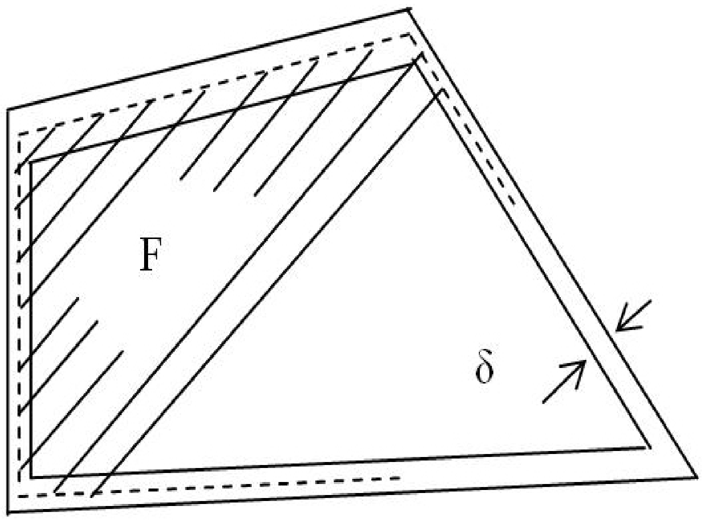
$$W_S = k_2 b^2 h$$



Coefficient k_1 and k_2 depend on b to h ratio (see table below)

h/b	k_1	k_2
1	0.141	0.208
1.5	0.196	0.231
2	0.229	0.246
3	0.263	0.267
6	0.298	0.299
....	....	....
∞	0.333	0.333

1.3 Thin Walled Members



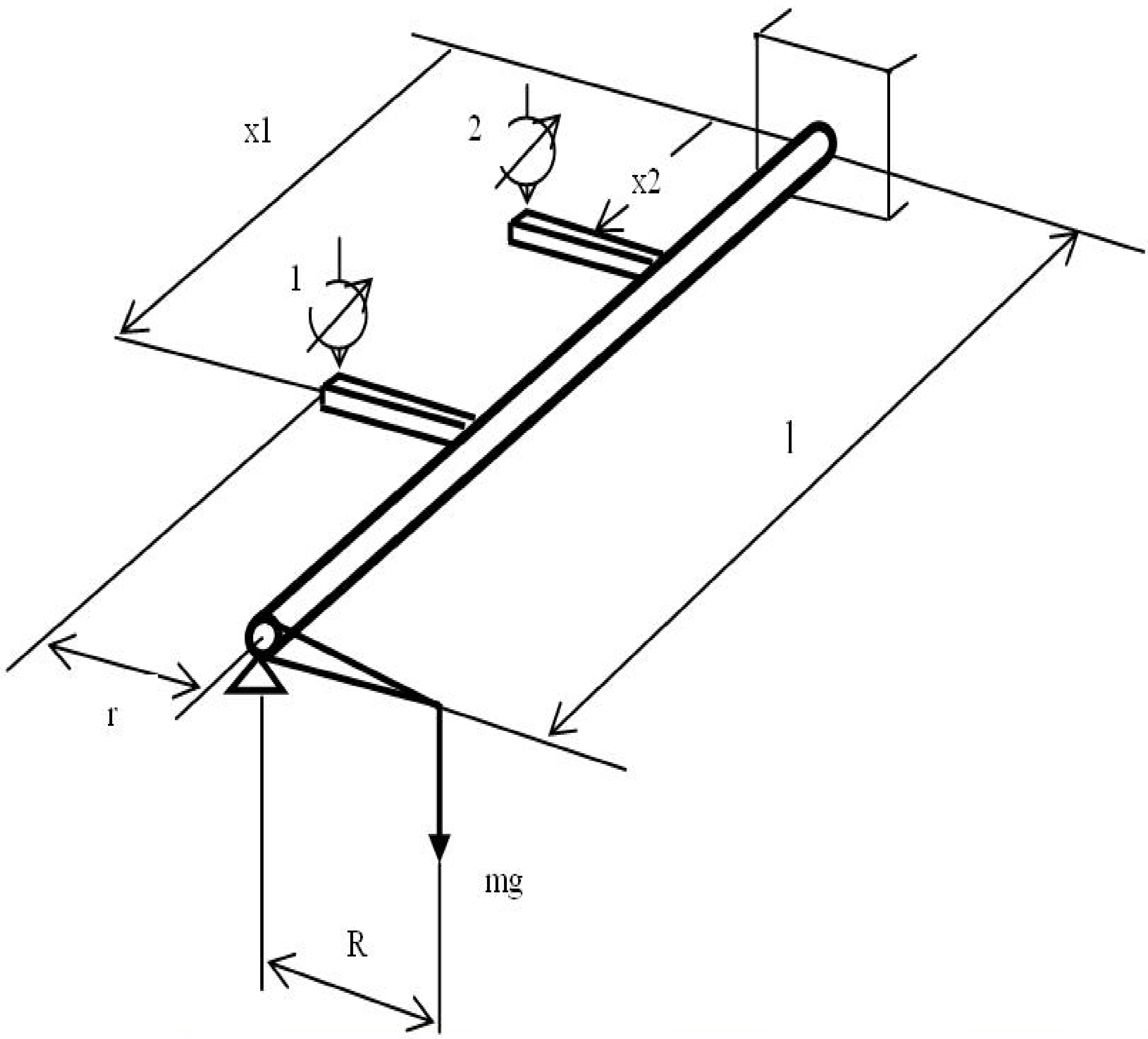
Stress distribution

$$\tau = \frac{M_T}{2F\delta}$$

where:
F - area surrounded by the middle line of the wall, δ - thickness

2 Exercise

2.1 Circular and Rectangular Rods



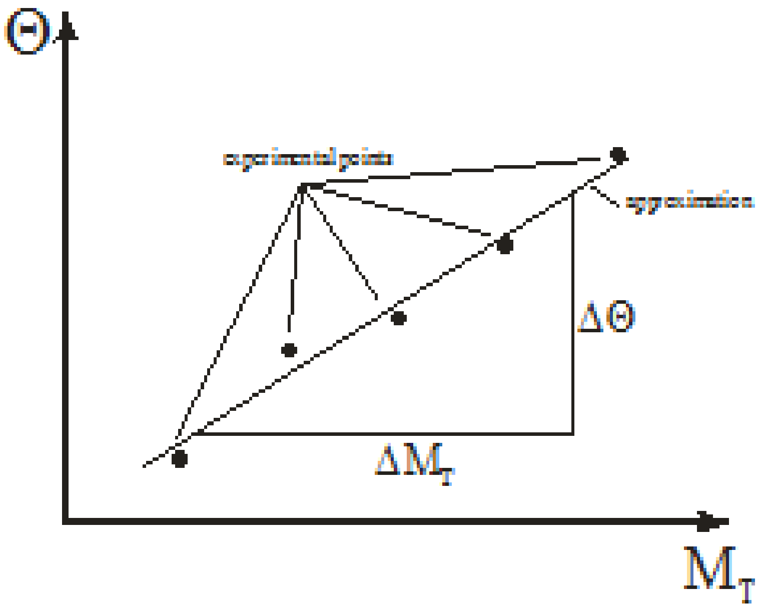
	m	f_1	f_2	φ_1	φ_2	$\Delta\varphi$	Θ	M_s
	kg	mm	mm	rad	rad	rad	rad/m	Nm
1								
2								
3								
4								
5								

$$\phi_1 = \frac{f_1}{r},\ \phi_2 = \frac{f_2}{r}$$

$$\Delta\phi = \phi_1 - \phi_2$$
$$\Theta = \frac{\Delta\phi}{x_1 - x_2}$$
$$M_T = mgR$$

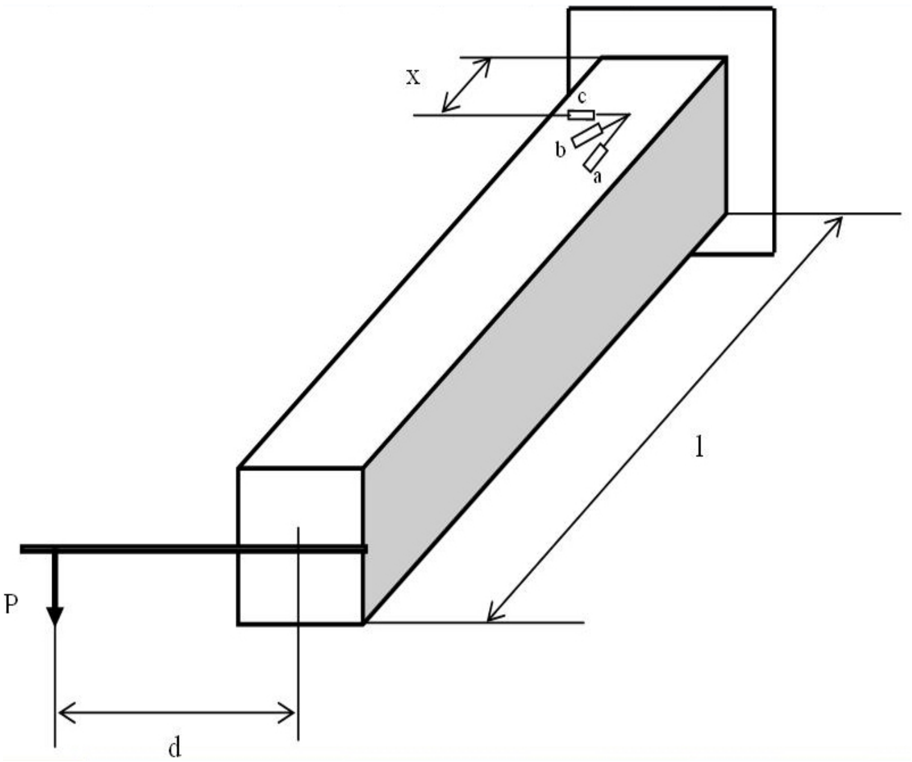
To calculate shear modulus:

1. draw $M_T = M_T(\Theta)$ diagram basing on the data in the table



2. read ΔM_T and $\Delta\Theta$ from the diagram and calculate shear modulus:
   $G = \frac{\Delta M_T}{\Delta\Theta}\frac{1}{J_0}$ for circular rod,
   $G = \frac{\Delta M_T}{\Delta\Theta}\frac{1}{J_S}$ for rectangular rod

2.2 Thin Walled Member

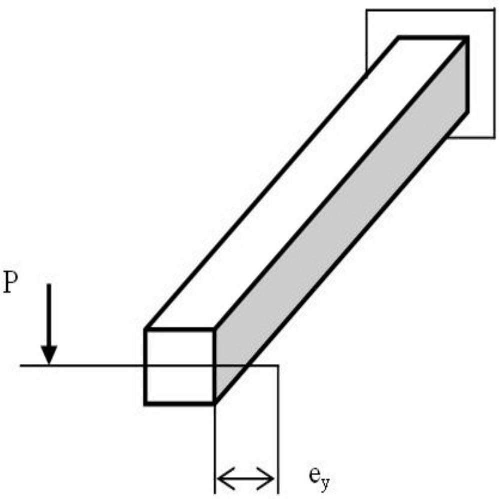


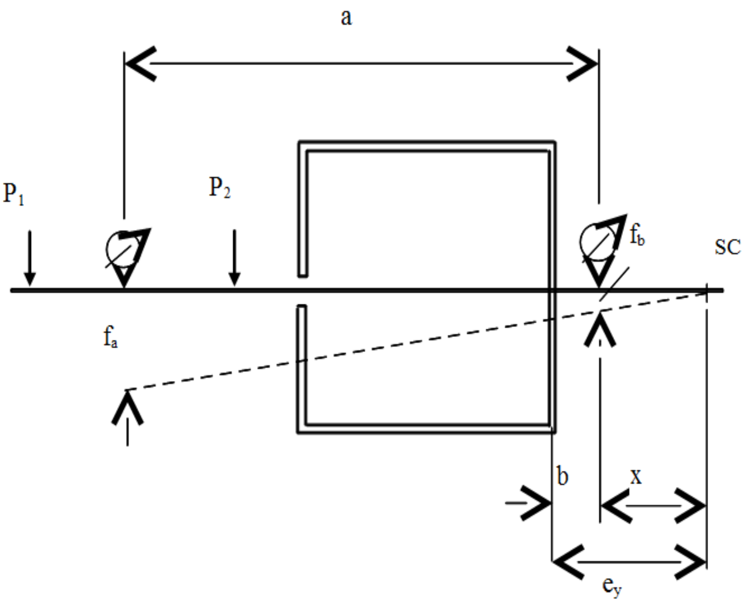
If $d \neq 0$ the member is bent and twisted simultaneously. In order to find strain for pure torsion one has to act as follows:

1. set $d \neq 0$, do measurements for bending+torsion (B+T),
2. set $d = 0$, do measurements for bending (B)
3. calculate strains for torsion as a difference between strains in B+T state and T state.

2.3 Shear Centre

For symmetrical cross-section shear centre (SC) is located on the axis of symmetry.





The broken line shows rotation of the cross-section around shear centre when the rod is loaded with a pair of forces ($P_2 = -P_1$). f_a and f_b are displacements measured by dial indicators. These values may be found indirectly by means of the following algorithm:

1. load the member with any force P at any point, measure displacements ${f_a}^1$ and ${f_b}^1$
2. load the member with the same force P but at different point, measure displacements ${f_a}^2$ and ${f_b}^2$
3. calculate displacements for pure torsion: $f_a = {f_a}^1$ - ${f_a}^2$ and $f_b = {f_b}^1$ - ${f_b}^2$

Shear centre location (e_y) may be calculated from the formula:

$$e_y = b + x$$

where: $x = \frac{a f_b}{f_a - f_b}$