Warsaw University of Technology

Faculty of Power and Aeronautical Engineering
WARSAW UNIVERSITY OF TECHNOLOGY

Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM2)

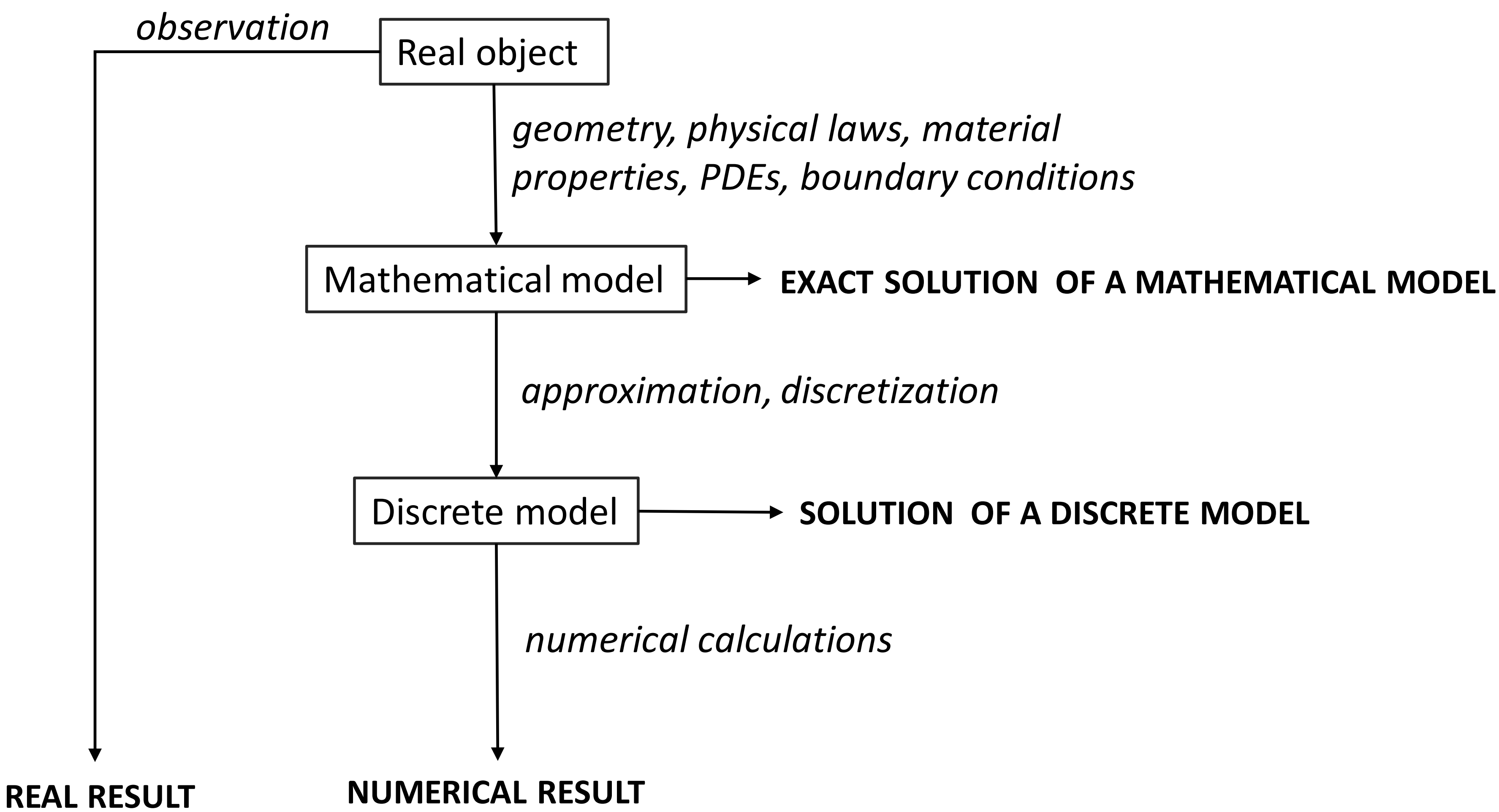
Introduction

The finite element method (FEM) is an approximate method which can be used as a numerical procedure to solve physical problems including:

- solid body mechanics,
- heat transfer,
- fluid flow,
- electromagnetism,
- coupled field problems
- …

FEM was developed in 1950s to solve problems for the civil and aeronautical industry. The method become the most powerful analysis tool, mainly due to the development of computers.

Modelling

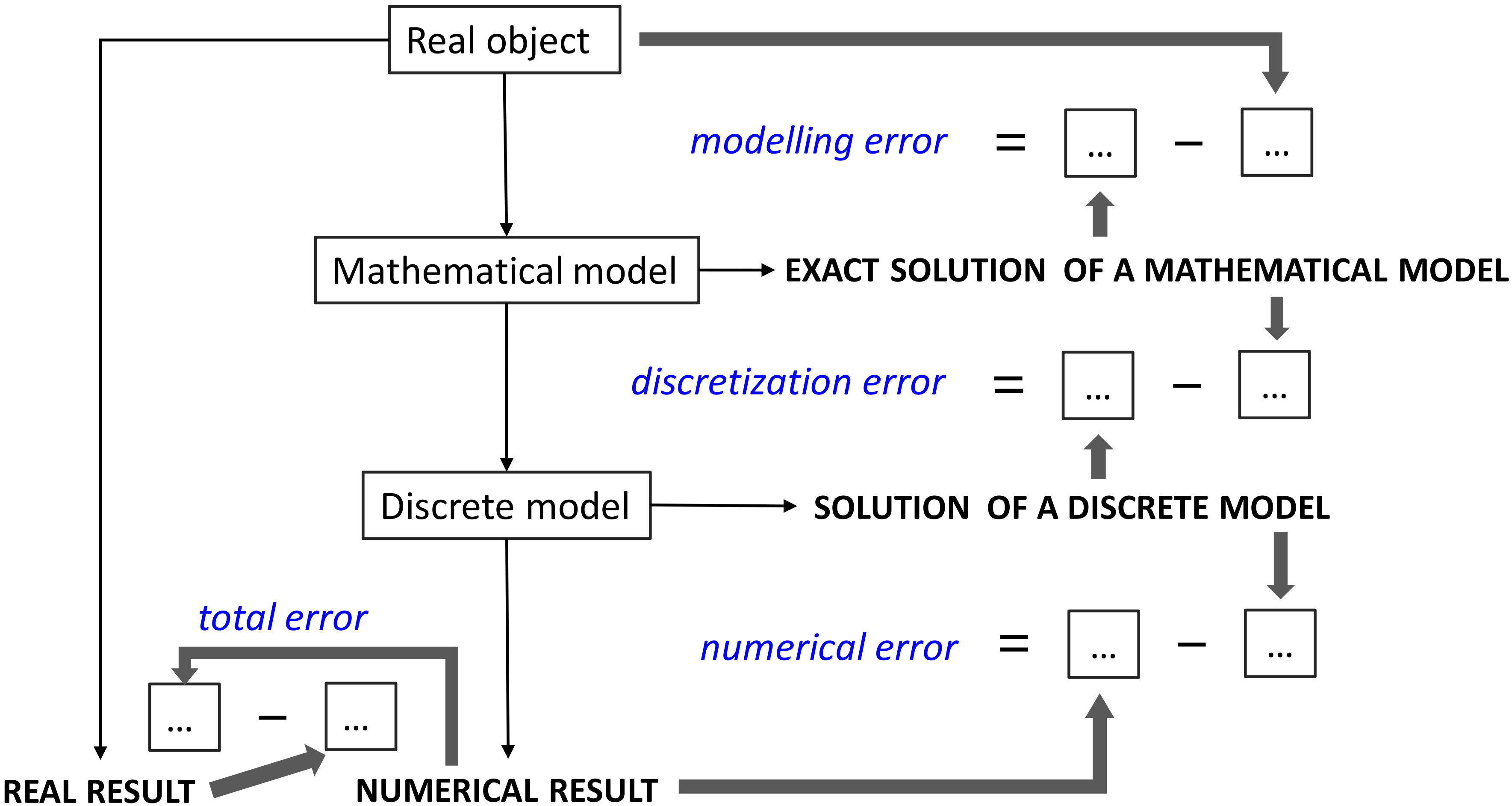


observation
Real object
geometry, physical laws, material properties, PDEs, boundary conditions
Mathematical model
EXACT SOLUTION OF A MATHEMATICAL MODEL
approximation, discretization
Discrete model
SOLUTION OF A DISCRETE MODEL
numerical calculations
REAL RESULT
NUMERICAL RESULT

Errors

total error = modelling error + discretization error + numerical error

modelling error ≈ discretization error ≈ numerical error → min



Modelling error

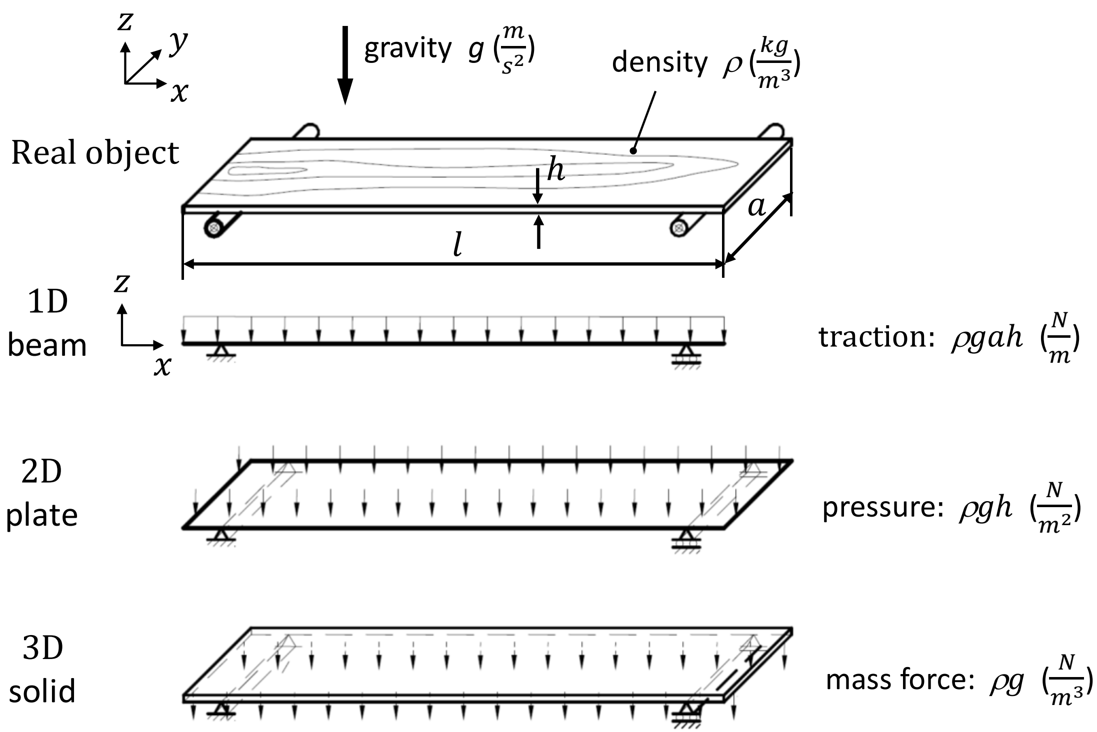
Available information about the real object:

- material data
- geometry
- work conditions

Simplifing assumptions

- dimensionality
- material model
- nonlinearities
- type of load

Example. Wooden board loaded by gravity.



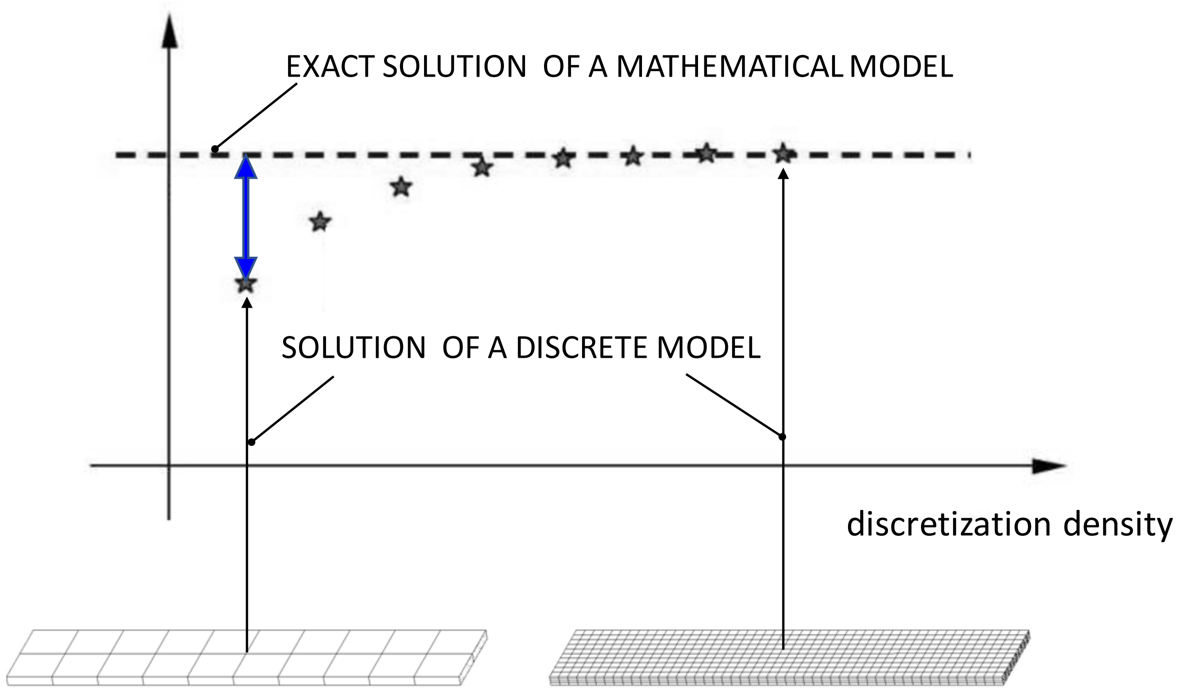
Discretization error

Discretization

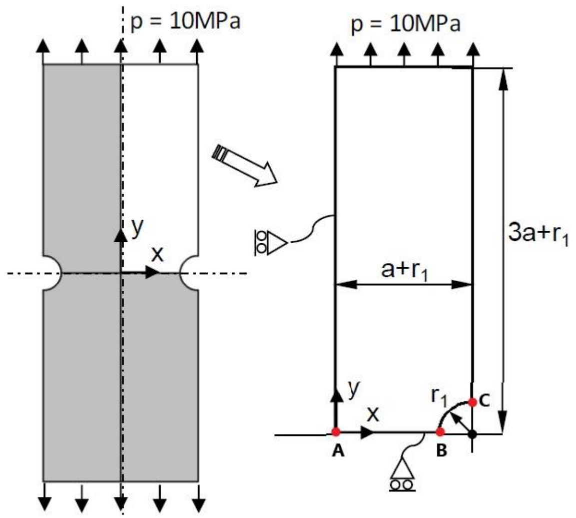
- type (mapped, free, sweep)
- density

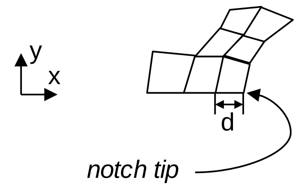
Finite element

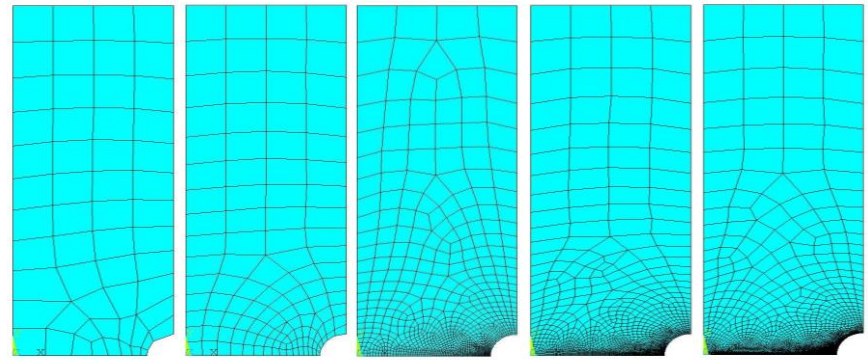
- shape functions
- integration scheme



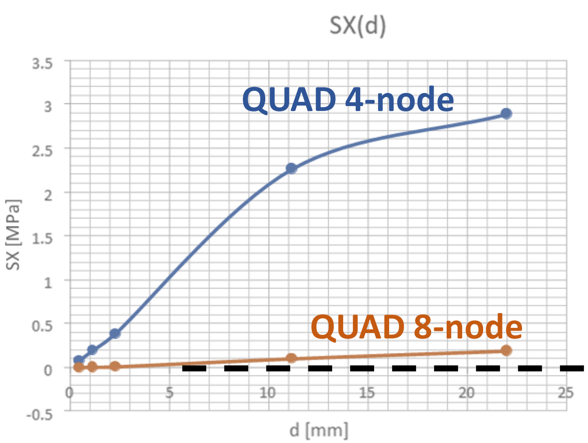
Example. Plate with notches

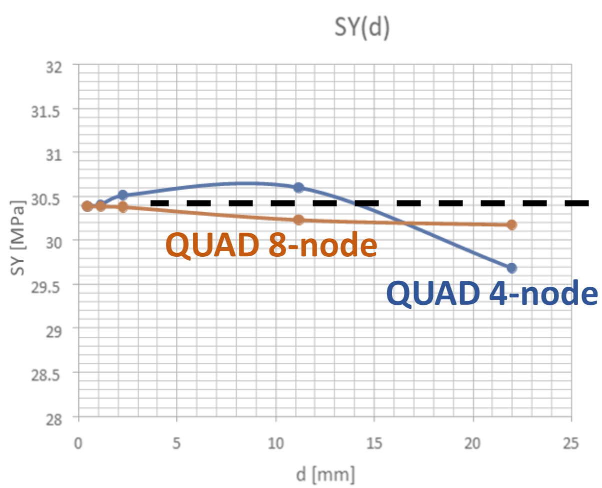


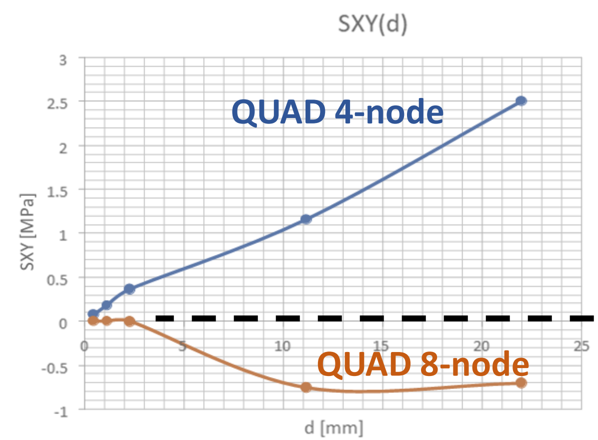




Stress components at the notch tip versus element size *d* (numerical results)







- - - - EXACT SOLUTION OF A MATHEMATICAL MODEL

Numerical error

- solver
- condition number

$$cond([K]) = \|K\| \cdot \|K\|^{-1}$$

- rounding (number of significant digits)

Approximately, if the condition number $cond([K]) = 10^k$, then up to *k* digits can be lost during solution of the system of linear equations.

$$r \geq p - \log_{10}\left(cond\left([K]\right)\right)$$

p – number of significant digits in the computer representation of numbers

r – number of significant digits of the result

In FE models the value of $cond([K])$ can reach 10^8

FE modeling – basic steps

Mathematical model

↓

Preprocessor

geometry, element types, material properties, approximation, discretization

↓

Discrete model

↓

Solution

analysis type, boundary conditions, solver, contents of a result file

↓ *numerical calculations*

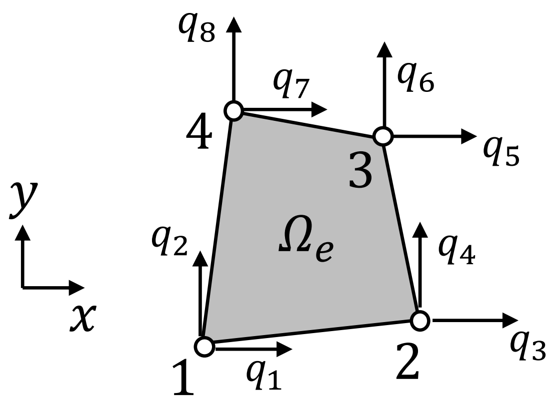
NUMERICAL RESULT

↓

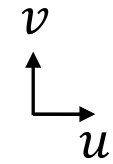
Postprocessor

numbers, graphs, contour maps, animations, **error estimation**

Example. DOF solution $u(x,y)$ for 2D problem. FE model with 4-node quadrilaterals

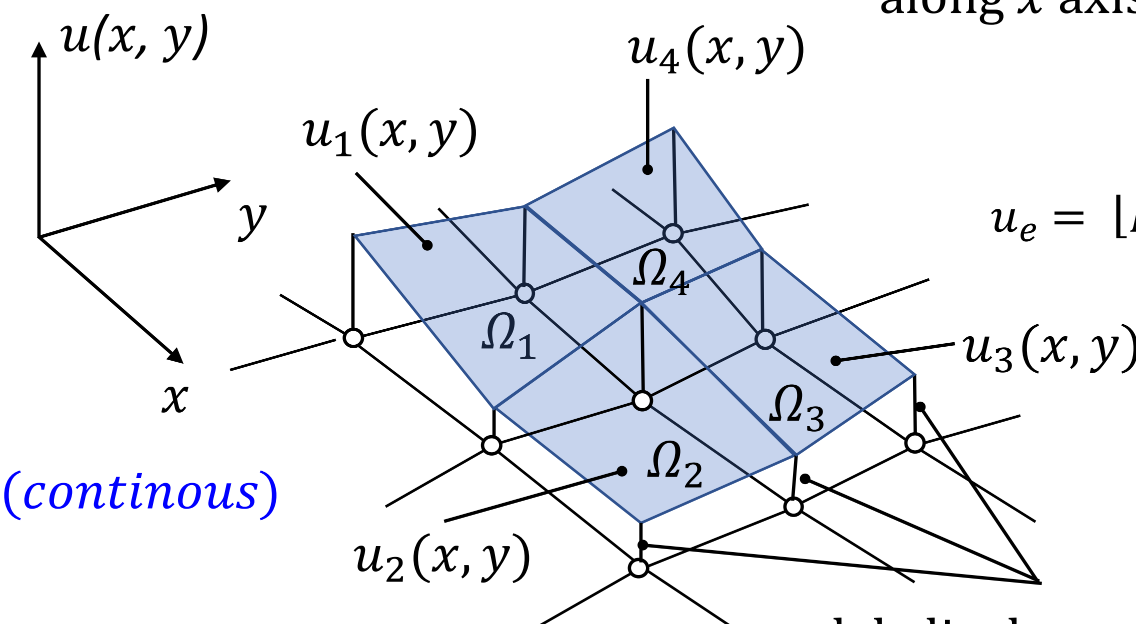


$$\underset{2\times 1}{\{u\}} = \underset{2\times 8}{[N]}\underset{8\times 1}{\{q\}_e}$$



$u_e(x,y)$ − displacement function along x axis

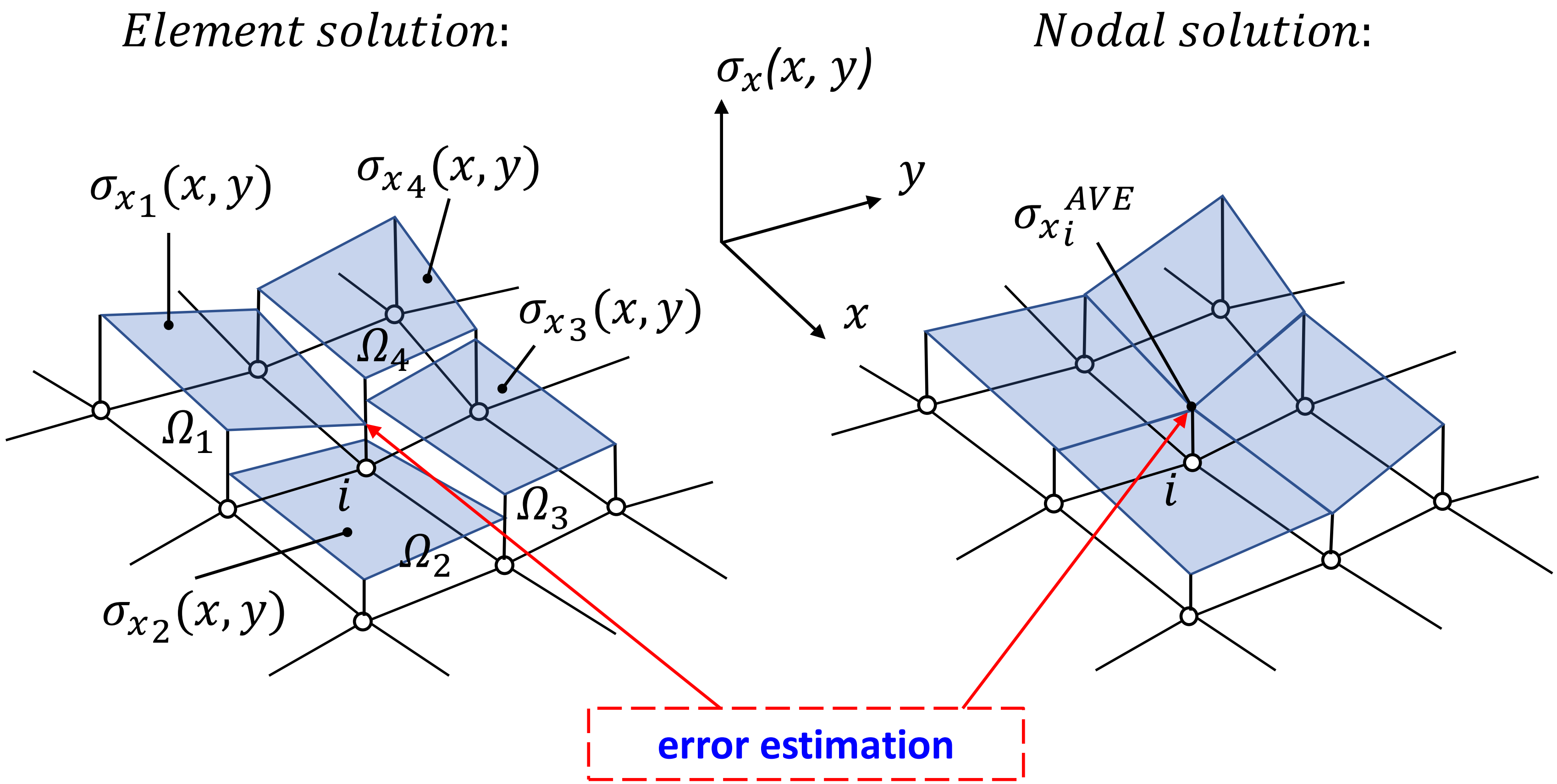
$$u_e = \lfloor N_1 \; 0 \;\; N_2 \; 0 \;\; N_3 \; 0 \;\; N_4 \; 0 \rfloor \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{Bmatrix}_e$$



(continous)

nodal displacements in x direction

Example. Stress component $\sigma_x(x,y)$ for 2D problem. FE model with 4-node quadrilateral elements



For $k = 4$:

$$\sigma_{x_i}^{AVE} = \frac{\sigma_{x_1}(x_i, y_i) + \sigma_{x_2}(x_i, y_i) + \sigma_{x_3}(x_i, y_i) + \sigma_{x_4}(x_i, y_i)}{4}$$