Integrated Laboratory

Strength of Materials and Structures

Bending

Before attending the laboratory students should recollect the following topics:
types of bending: pure, transverse, skew (unsymmetrical), internal forces distribution, stress distribution, radius of curvature, deflection, Hook's law, superposition rule, Betti's theorem, de Saint Venant principle. plane cross-section hypothesis

Recommended Bibliography:

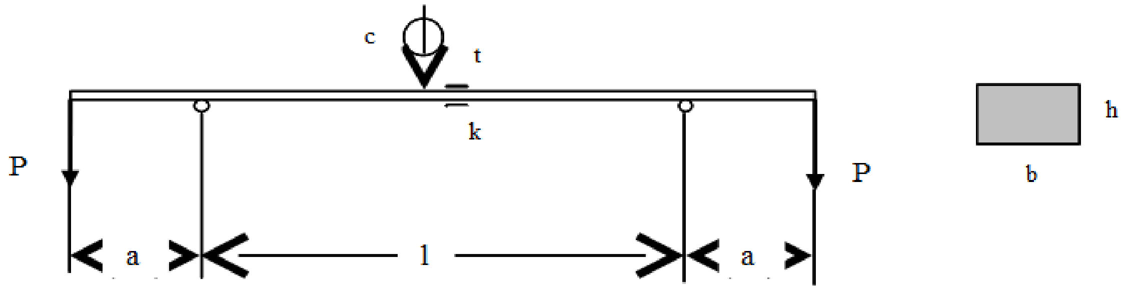
- William A. Nash *Strenght of materials*
- Roy R. Craig *Mechanics of Materials*
- *Mechanika Materiałów i Konstrukcji* edited by Marek Bijak-Żochowski
- Own lecture notes
- the Internet (for lazy students)

1 Purpose of the Exercise

Purpose of the exercise is experimental verification of the basic mechanics of structures rules and principles and widening knowledge on behaviour of bars during bending. Different types of bending (i.e. pure, transverse, and skew) are investigated. Dial indicators and strain gauges are used to perform measurements.

2 Pure Bending

2.1 Experimental stand



Picture 1: *Experimental stand*

Two symmetrically spaced pivots support the beam. The beam is loaded at both ends with the same force P. Thus there is pure bending state between supports. A dial indicator “c” is located right in the middle between the supports. Next to it two active strain gauges (“t” and “k”) are applied (on the upper and the lower surface of the beam). They are connected into the one Wheatstone’s bridge circuit.

2.2 The Exercise

1. read the values indicated by the dial indicator (c_i) and the bridge (i_i) before loading ($i = 0$)

2. read the values indicated by the dial indicator (c_i) and the bridge (i_i) for different loads ($i = 1, 2, ...$)

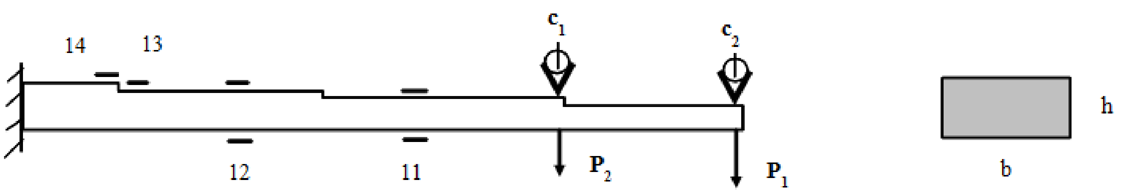
i	P	c_i	$f_C=c_i-c_0$	i_i	$\varepsilon = \frac{1}{2}\frac{k_B}{k_T}(i_i - i_0)$[1]
	kG	mm	mm	‰	‰
0	0		---		---
1					
2					
3					

2.3 Result study

1. Calculate deflection f_C and strain for outer layers of the beam ϵ for different loads P_i, $(i = 1, 2, ...)$.
2. Calculate Young modulus value basing on (a) the deflection f_C, (b) the strain ϵ and compare with values which may be found in the books. Estimate relative error.
3. Calculate radius of curvature of the beam basing on (a) the deflection f_C, (b) the strain ϵ and compare with theoretical value. Estimate relative error.
4. . Compare deflection f_C with theoretical value. Estimate relative error.

3 Transverse Bending

3.1 Experimental Stand



Picture 2: *Experimental stand*

The cantilever beam may be loaded with forces P_1 and P_2 (separately or at the same time). At the points where the load may be applied two dial indicators ("c1" and "c2") are installed. Pairs of active strain gauges („t" and „k") are applied in cross-section „11" and „12" whereas there are single active strain gauges in cross-sections "13" and "14".

3.2 The Exercise

1. read the values indicated by the dial indicators and the bridge for all cross-sections before loading.

c_1^0 [mm]	c_2^0 [mm]	i_{11}^0 [‰]	i_{12}^0 [‰]	i_{13}^0 [‰]	i_{14}^0 [‰]

2. read the values indicated by the dial indicators for forces P_1 and P_2 acting separately and at the same time and calculate deflections f_{C1} and f_{C2}.

	P_1 [kG]	P_2 [kG]	c_1 [mm]	c_2 [mm]	$f_{C1}=c_1-c_1^0$ [mm]	$f_{C2}=c_2-c_2^0$ [mm]
$P_1=0, P_2\neq 0$	0					
$P_1\neq 0, P_2=0$		0				
$P_1\neq 0, P_2\neq 0$						

3. read the values indicated by the bridge for different values P1 force and calculate corresponding strain values.

i	P_1 [kG]	i_{11}^i [‰]	i_{12}^i [‰]	i_{13}^i [‰]	i_{14}^i [‰]
1					
2					
3					

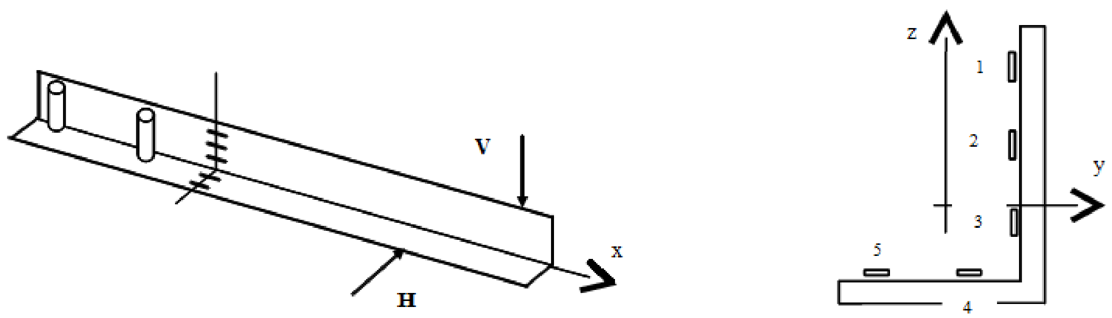
i	$\varepsilon_{11} = \frac{1}{2}\frac{k_B}{k_T}\left(i^i_{11} - i^0_{11}\right)$ [°/oo]	$\varepsilon_{12} = \frac{1}{2}\frac{k_B}{k_T}\left(i^i_{12} - i^0_{12}\right)$ [°/oo]	$\varepsilon_{13} = \frac{k_B}{k_T}\left(i^i_{13} - i^0_{13}\right)$ [°/oo]	$\varepsilon_{14} = \frac{k_B}{k_T}\left(i^i_{14} - i^0_{14}\right)$ [°/oo]
1				
2				
3				

3.3 Result Study

1. verify the superposition rule (basing on point 3.2.2)
2. verify Betti's theorem (basing on point 3.2.2)
3. verify de Saint Venant principle (compare experimental and theoretical values of stress in cross-section „11", „12", „13" and „14")
4. calculate Young modulus basing on measurements for cross-section "11" and compare it with values which may be found in the books. Estimate relative error.
5. calculate Young modulus basing on measurements for cross-section "12" and compare it with values which may be found in the books. Estimate relative error.

4 Skew (Unsymmetrical) Bending

4.1 Experimental stand



Picture 3: *Experimental stand*

The beam is supported by two pinned bearings and may be loaded with vertical force V and horizontal force H (at the same time or separately). Five strain gauges (no. 1 to 5) are applied at the same cross-section.

4.2 The Exercise

1. read the values indicated by the bridge before loading.

i_1^0 [‰]	i_2^0 [‰]	i_3^0 [‰]	i_4^0 [‰]	i_5^0 [‰]

2. read the values indicated by the bridge after loading and calculate corresponding strain values.

i_1 [‰]	i_2 [‰]	i_3 [‰]	i_4 [‰]	i_5 [‰]

$\varepsilon_1 = \frac{k_B}{k_T}\left(i_1 - i_1^0\right)$ [‰]	$\varepsilon_2 = \frac{k_B}{k_T}\left(i_2 - i_2^0\right)$ [‰]	$\varepsilon_3 = \frac{k_B}{k_T}\left(i_3 - i_3^0\right)$ [‰]	$\varepsilon_4 = \frac{k_B}{k_T}\left(i_4 - i_4^0\right)$ [‰]	$\varepsilon_5 = \frac{k_B}{k_T}\left(i_5 - i_5^0\right)$ [‰]

4.3 Result study

1. Calculate experimental values of stress at points where strain gauges are applied.
2. Calculate theoretical values of stress at the edges of the cross-section where strain gauges are applied.

$$\sigma(y,z) = -\frac{m_y}{j_y}z - \frac{m_z}{J_z}y$$

$$m_y = \kappa(M_y - M_z\frac{J_{yz}}{J_z})$$

$$m_y = \kappa(M_z - M_y\frac{J_{yz}}{J_y})$$

$$\frac{1}{\kappa} = 1 - \frac{{J_{yz}}^2}{J_y J_z}$$

"y" and "z" are coordinates of points in system with the origin in the centre of the cross-section.

3. Draw diagram which shows theoretical stress distribution along the edges. Mark experimental values as points.
4. Compare theoretical and experimental values of stress. Estimate relative error.
5. Calculate neutral axis ($\sigma = 0$) orientation.