FUNDAMENTALS OF RELIABILITY ANALYSIS



Risk in M-T-E system



 $\begin{pmatrix} risk \\ measure \end{pmatrix} = \begin{pmatrix} unreliability \\ measure \end{pmatrix} \cdot \begin{pmatrix} hazard \\ measure \end{pmatrix}$

Basic terms and concepts

Undesirable event is an event, which occurrence, in the considered **M-T-E** system, could result in hazard exposure for humans or property.

Failure is an undesirable event occurring in the M-T sub-system.

The unit's **Failure** is an event disabling physical or agreed upon performance of specific functions.



Defect, fault, damage



Reliability

Reliability – ability of a unit to function properly during a specified period of time without failure.



A M-T System

A man (e.g. operator)

A group of people

A technical object

An assembly of a technical object

A component of an object



Reliability

characteristics in the unit under investigation of the ability to achieve specified requirements under given conditions in specified time



"Reliability is quality based on time"





State of the unit



functioning state, failure state not functioning

Functioning time Regenerating time Service life





Course of a unit operation process

non-reparable unit





Course of a unit operation process

reparable unit





Functioning time $-\tau$ Regenerating time $-\vartheta$ Service life $-t_{rw}$

RELIABILITY MEASURES

Reliability function (survival probability)

 $R(t) = P\{T > t\}$

 p_{min}



Examples of unit's functioning time until failure





Failure rate



Failure rate expresses the proneness of the unit to fail at time (age) t



Failure rate λ is the probability of failure in small time interval (a unit of time) following the time t, assuming that in time t the unit is functioning







Failure rate



Conditional probability



Failures per unit time referred to remaining products



Typical failure rate



"Bath-tub shape"



Failure intensity of a technical object (unit)

Failure phases



Exponential distribution



Probability density curve for $\lambda = \text{const.}$



 $\lambda(t) = \frac{f(t)}{R(t)} = \frac{dQ(t)}{dt} \cdot \frac{1}{R(t)} = \frac{d\left[1 - R(t)\right]}{dt} \cdot \frac{1}{R(t)} =$ $= -\frac{dR(t)}{dt} \cdot \frac{1}{R(t)} = -\frac{d}{dt} \cdot \ln R(t)$ $R(t) = e^{-\int_{0}^{t} \lambda(\tau) d\tau}$

 $R(t)=e^{-\lambda t}$



Example





Failure rate of a gearbox is constant (the unit is not ageing) and is 10^{-5} [1/h].

Service life of the gearbox 10 000 [h]. What is the probability of the gearbox functioning without failure in:

- First 100 h
- Service life
- Last 100 h

0,999 0,9048 0,999

 $R(t) = e^{-0,00001100} = e^{-0,001} = 0,999$



Estimator of failure rate



n(*t*) – sample size in time *t*

 $b(t,t+\Delta t)$ – number of units, that failed in period $(t,t+\Delta t)$

A course of failure rate



Death intensity (fatality rate) for **men** (— 1978, --- 1994) and **women** (— 1978, 1994) in Poland



Time-to-failure

T - random variable,

in reliability analysis time-to failure







Statistically expected lifetime (service life)

(mean service life)

$$E\boldsymbol{T} = \int_{0}^{\infty} t \cdot f(t) dt = \int_{0}^{\infty} R(t) dt$$

for non-reparable units ET is called Mean Time To Failure

for **reparable** units

ET is called Mean Time Between Failures

Reliability measures characteristic for reparable units

Availability – important feature of hazard counteraction systems (e.g. rescue system)



Steady-state Availability A – probability that a unit will be available (in functioning state) when required

$$A = \frac{ET}{ET + E\theta}$$

 $ET i E\theta$ – expected (mean) values for the state of functioning and the state of failure

Limiting availability

For *i*th component (unit)



 $A_i = \frac{MTBF_i}{MTBF_i + MTTR_i}$

MTBF – Mean Time Between Failures*MTTR* – Mean Time To Repair

 $Availability = \frac{Relaiability}{Relaiability + Maintainability}$



Exercise

It has been observed, that mean time of service for a device is 12 years. It is known that the failure rate for this device is constant.

Find the following:

- reliability of the device in 5 years
- time, when the reliability drops to 0,5





(Systems reliability models) RELIABILITY STRUCTURE

Reliability structure defines dependence of the unit's reliability from reliability of its components

Methods of reliability structure presentation:

- block diagrams
- fault trees



RELIABILITY BLOCK DIAGRAMS

Basic reliability structures:

- > series
- > parallel
- ▶ "k-out-of-m"
- redundancy





Series structure



$$T, T_i (i = 1, 2, ..., m)$$

 $\mathbf{R}(\mathbf{t}) = \prod_{i=1}^{m} \mathbf{R}_{i}(t)$



Examples: ..., worker-lathe, group of people doing particular task, ...

Example of a series structure

Fails when the first component fails

Example:

5 identical light bulbs with $ET_i = 100 h$

ET = 100/5 = 20 h





 $ET = \frac{ET_i}{m}$



Parallel structure



if random variables T_i are independent

$$R(t) = 1 - \prod_{i=1}^{m} [1 - R_i(t)] \qquad \qquad Q(t) = \prod_{i=1}^{m} Q_i(t)$$

Fails when all components fail

Example of a parallel structure



Example:

5 identical light bulbs with $ET_i = 100 h$

ET = 100(1+1/2+1/3+1/4+1/5) = 228 h

$$ET = ET_i \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{m} \right)$$



Structure with redundancy

Standby redundancy



$$\mathbf{T} = \sum_{i=1}^{m} \mathbf{T}_{i}$$

Example of a redundant structure

Example:

5 identical light bulbs with $ET_i = 100 h$

ET = 5*100 = 500 h





Structure with redundancy

Active redundancy = parallel system



Reduced redundancy

Examples: ...; systems, where failures may cause considerable damage (large chemical installations, nuclear plants, rescue systems)



Exercise

Reliability of an unmanned aerial vehicle (aircraft) have to exceed 0.99 during the mission time of 10 hours.

What should be the mean time to failure (MTTF), if the exponential distribution of T is assumed?

 $R(10) \ge 0.99$







Exercise

Series reliability system is composed of identical items. At time t_k reliability of the items is $R(t_k)=0.96$. How many elements could be in the system, when it is required that the system reliability is $R_s(t_k)>=0.8?$

m = ?

m = 5.47, m = 5


FUNDAMENTALS OF RELIABILITY ANALYSIS

MODELLING

Modelling procedure of an item reliability structure

1. Decomposition of the item



decomposition depth



assemblies sub-assemblies components

fragments of components

- 2. Determination of the item failure criterion (definition of failure)
- 3. Selection of a reliability structure



TREE METHODS IN RISK ANALYSIS



unreliability \rightarrow risk



unreliability → fault tree hazard → event tree



Fault tree method

Fault tree – logical diagram which shows the relation between item failure, i.e. a specific undesirable event in the system, and failures of the components of the system



The most important symbols of events and gates:

a) "or" gate, b) "and" gate, c) basic (input) event, d) description of event ("comment" rectangle), e) transfer symbol, f) undeveloped event



AND gate

logical product



The output event A occurs if all input events (faults) occur

$$q = q_1 \cdot q_2 \cdot \ldots \cdot q_n$$





logical sum



The output event A occurs if at least one of the input events (faults) occurs

$$q = 1 - (1 - q_1)(1 - q_2) \dots (1 - q_n)$$

$$q_i(t) = 1 - \dot{R}_i(t)$$

if $q_i(t) \ll 1$, then

$$q \cong q_1 + q_2 + \ldots + q_n$$





of a fault tree construction



chemical installation

- 1 temperature sensor
- 2 thermo regulator
- 3 wire connection

Choice of the top event (undesirable event) $A^{(k)}$, which probability $Q^{(k)}(1)$ is to be evaluated

 $\mathbf{A}^{(k)} = (\mathbf{T} > T_{acc})$

Example of a fault tree





The fault tree continued







description of the reliability structure

Advantages:

- both graphical and verbal information
- contains undesirable events, that can occur in the system and its components
- more detailed description of the system reliability

The fault tree method is applied in:

- qualitative risk analysis, carried out to eliminate causes of failures,
- quantitative reliability evaluation and analysis,
- evaluation and analysis of risk

likelihood of events on the tree lowest level

Event tree method



Event tree – a diagram showing chronological chain of events, important for the system functioning, occurring after a chosen event (fault).

Hazard modelling using event trees



 ρ_k – number of the event tree sequences, q^(kv) – probability of the *v*-th sequence occurrence



The example of top event probability calculation (continued)

Values needed:

e.g. *p*₁,*p*₂, *p*₃, *p*₄

Sources of data collection:

- statistic methods
- expert methods





literature internet accident databases

The example of top event probability calculation (continued)



 $Q(1) = p_1 + p_2 p_3 + p_4 = 8 \cdot 10^{-3} + 0.5 \cdot 10^{-3} + 20 \cdot 10^{-3} = 28.5 \cdot 10^{-3}$

Use of tree methods in risk modelling and analysis



An example of tree method applied to modelling and analysis of flood risk





METHODS FOR RELIABILITY EVALUATION AND ANALYSIS

Methods based on reliability models may be used in analysis of the following reliability systems:

- undesirable events prevention
- hazard counteraction
- rescue

E.g. simulation of different alternative actions during a rescue operation to evaluate their reliability etc.

RELIABILITY MODELLING

Reliability model of an item – a system with respect to reliability representing the real item and replacing it in planned reliability analyses

- 1. Model of reliability structure
- in the form of block diagram



 $R(t) = R_4(t) \cdot \{1 - [1 - R_3(t)][1 - R_1(t)R_2(t)]\}$

in the form of fault tree



RELIABILITY MODELLING cont.

2. Human reliability models

3. Phenomenon models, that may lead to failure





Methods based on models

- Used mainly for analysis of the effects of different factors on the modelled item reliability. Allows evaluation of factors taken into account in the model.
- They are more difficult then statistical or expert methods
- Could be used for reliability analysis of systems, where hazards are big



METHODS FOR RELIABILITY EVALUATION AND ANALYSIS





CHOICE OF THE RELIABILITY LEVEL

Cost-reliability dependence



RELIABILITY LEVEL

Service life of home appliances with electric motors

appliance	service life in years	work Ioad in 1 year in [h]	MTTF in hours
coffee grinder	10	5-10	200
lawn mover	10	20-50	500
washing machine	10	30-200	3000
cooling fan	5	10-600	3000



REPARABLE ITEMS

Repair strategies:

replacement/repair at failures
preventive maintenance



The aim of preventive maintenance:

to reduce the occurrence of the item failure

Periodical testing/replacement

The effect of preventive maintenance on item's reliability (availability)



- Analysis of effectiveness
- > Optimisation of preventive maintenance strategy

FUNDAMENTALS OF RELIABILITY ANALYSIS







Failure mode and effect analysis

Basic principles for FMEA

FMEA is a simple analysis method to reveal possible failures and to predict the failure effects on the system as a whole.

The method is inductive; for each component of the system we investigate what will happen if this component fails. The method represents a systematic analysis of the components of the system to identify all significant failure modes and to see how important they are for the system performance.

Only one component is considered at a time, the other components are then assumed to function perfectly. FMEA is not suitable for revealing critical combinations of component failures.

SYSTEM/EQUIPMENT:

EXECUTED BY:

REF. DIAGRAM/DRAWING NO.:

DATE:

Identifi- cation	Function/ operational	Failure mode	Effect on other units	Effect on the system	Correc- tive	Failure frequency	Failure effect	Remarks
1	2	3	4 t	5	6	7	8	9

Identification (column 1). Here the specific component is identified by a description and/or number. It is also possible to refer to a system drawing or a functional diagram.

SYSTEM/EQUIPMENT:

EXECUTED BY:

REF. DIAGRAM/DRAWING NO.:

DATE:

Identifi- cation	Function/ operational state	Failure mode	Effect on other units in the system	Effect on the system	Correc- tive measures	Failure frequency	Failure effect ranking	Remarks
1	2	3	4 5	5	6	7	8	9

Function, operational state. The function of the component, i.e. its working tasks in the system, is briefly described.

The state of the component when the system is in normal operation is described, e.g. whether it is in continuous operation mode or in stand-by mode. *Example:* In a chemical process plant a specific valve is considered as a component in the system. The function of the valve is to open and close at demand. "The valve does not open at demand" and "the valve does not close at a demand" are relevant failure modes, as well as "the valve opens when not intended" and "the valve closes when not intended". However, "washer bursts" is an example of the cause of a specific failure mode.

Failure modes. All the possible ways the component can fail to perform its function are listed under this column. Only the failure modes that can be observed from "outside" are included. The internal failure modes are to be considered as causes of failure. These causes can possibly be listed under a separate column. In some cases it will also be of interest to look at the basic physical and chemical processes that can lead to failure (failure mechanisms), such as corrosion. Often we also state how the different failure modes of the component are detected, and by whom.

SYSTEM/EQUIPMENT:

EXECUTED BY:

REF. DIAGRAM/DRAWING NO.:

DATE:

Identifi- cation	Function/ operational state	Failure mode	Effect on other units in the system	Effect on the system	Correc- tive measures	Failure frequency	Failure effect ranking	Remarks
1	2	3	4 5	5	6	7	8	9

Effect on other units in the system. In those cases where the specific failure mode affects other components in the system it is stated in this column.

Emphasis should be given to identification of failure propagation which does not follow the functional chains of the functional diagrams.

For example: increased load on the remaining pillars that are supporting a common load when a pillar collapses; vibration in a pumping house may induce failure of the driving unit of the pump, etc.

SYSTEM/EQUIPMENT:

EXECUTED BY:

DATE

REF. DIAGRAM/DRAWING NO.:

			0					
Identifi- cation	- Function/ operational state	Failure mode	Effect on other units in the system	Effect on the system	Correc- tive measures	Failure frequency	Failure effect ranking	Remarks
1	2	3	4	5	6	7	8	9

Effect on system. In this column we describe how the system is influenced by the specific failure mode.

The operational state of the system as a result of failure, is to be stated, for example, whether the system is in the operational state, changed to another operational mode, or not in an operational state.

SYSTEM/EQUIPMENT:

EXECUTED BY:

DATE:

REF. DIAGRAM/DRAWING NO.:

Identifi- cation	Function/ operational state	Failure moo	e Effect on other units in the syster	Effect on the syster n	Correc- tive measures	Failure frequency	Failure effect ranking	Remarks
1	2	3	4	5	6	7	8	9

Corrective measures. Here we describe what has been done or what can be done to correct the failure, or possibly to reduce the consequences of the failure. We may also list measures that are aimed at reducing the probability that the failure will occur.
FMEA table

DEE DIACDAM/DDAM/INC NO .

SYSTEM/EQUIPMENT:

EXECUTED BY:

DATE.

			0		DAIL.			
Identifi- cation	- Function/ operational state	Failure mode	Effect on other units in the system	Effect on the system	Correc- tive measures	Failure frequency	Failure effect ranking	Remarks
1	2	3	4	5	6	7	8	9

Failure frequency. Under this column we state the estimated frequency (probability) for the specific failure mode and consequence. Instead of presenting frequencies for all the different failure modes, we may give a total frequency and relative frequencies (in percentages) for the different failure modes.

Exemplary failure effects:

Small: A failure that does not reduce the functional ability of the system more than normally is accepted.

Significant: A failure that reduces the functional ability of the system beyond the acceptable level, but the consequences can be corrected and controlled.

Critical: A failure that reduces the functional ability of the system beyond the acceptable level and which creates an unacceptable condition, either operational or with respect to safety.

Failure effect ranking. The failure is ranked according to its effect, with respect to reliability and safety, the possibilities of mitigating the failure, the length of repair time, production loss, etc.

FMEA table

SYSTEM/EQUIPMENT:

EXECUTED BY:

DATE:

REF. DIAGRAM/DRAWING NO.:

Identifi- cation	Function/ operational state	Failure mode	Effect on other units in the system	Effect on the system	Correc- tive measures	Failure frequency	Failure effect ranking	Remarks
1	2	3	4 5	5	6	7	8	9

Remarks. Here we state for example assumptions and suppositions.



An example of FMEA

Storage tank



An open container for preliminary storage of fluid for use in the production process.

The consumption of fluid in the process is not constant, and the liquid level in the tank will therefore vary.

Filling the tank is automatically controlled.



SYSTEM/EQUIPMENT: Storage tank EXECUTED BY: TAV

REF. DIAGRAM/DRAWING.NO.: DATE: 08.08.90 PAGE: 1 OF: 2

ldentifi- cation	Function/ operational state	Failure mode	Effect on other units in the system	Effect on the system	Failure frequency	Failure effect ranking
1	2	3	4	5	7	8
LSH	Switch that sends stop signal to V1 if the liquid level is high	Does not send signal when the liq. level is high	V1does not dose	The liquid level may increase abnormally	1% of total number of demands	3
		Sends signal when the liquid level is not high	V1 closes when not intended	The fluid supply stops	Once per year on average	1
LSHH	Switch that sends stop signal to V2 and open signal to V3 if the liquid level is abnormally high	Does not send signal when the liquid level is abnormally high	V2 does not dose. V3 does not open	The tank is over filled if V1 does not close	1% of total number of demands	4
		Sends signal when the liquid level is not abnormally high	V2 closes when not intended. V3 opens when not intended	The tank is drained	Once every 2nd year on average	2
V1	Stops the fluid supply when the liquid level is high. The valve is normally open	Does not dose at a signal		The liquid level may increase abnormally	2% of total number of demands	3
		Closes when not intended		The fluid supply stops	Once in 10 years on average	1
		Significant leakage		The fluid supply stops	Once in 10 years on average	1



Storage tank



SYSTEM/EQUIPMENT: Storage tank EXECUTED BY: TAV

REF. DIAGRAM/DRAWING.NO.: DATE: 08.08.90 PAGE: 2 OF: 2

ldentifi- cation	Function/ operational	Failure mode	Effect on other units	Effect on the system	Failure frequency	Failure effect
	state		in the system	-		ranking
1	2	3	4	5	7	8
V2	Stop the supply	Does not dose		Undesired supply	2% of total	2
	level is abnormally high. The valve is normally open	at a signal		The fluid is drained if V3 opens	demands	
		Closes when not intended		The fluid supply stops	Once in 10 years on average	1
		Significant leakage		The fluid supply stops	Once in 10 years on average	1
V3	Drain the fluid when the liquid level is abnormally high The valve is normally dosed	Does not open at a signal		Undesired supply to the storage	2% of total number of demands	3
		Opens when not intended		The fluid is drained	Once in 10 years on average	2
		Significant leakage		The fluid supply stops. The fluid is drained	Once in 10 years on average	1, 2



FMECA



Failure Modes, Effects and Criticality Analysis

By comparing failure frequency (probability) and failure effect (consequence) the criticality of the specific failure mode can be determined

Probability/frequency

Consequence category Small Significant Critical

Very unlikely, Once per 1000 year or more infrequently

Unlikely, Once per 100 year Quite likely, Once per 10 year

Likely, Once per year

Frequently, Once per month or more frequently

Example o Design FMEA

Revision	6.0 2/11/9	8				[De	sign FMEA	1																
	System Subsystem	Customer Chrysier Mo	tors Corporation					Customer Part No. DC-77323-XYZ						Org. Date 2/11/98	•	Pag 1 of	e 2								
iti	Component	Supplier Any Compa	ny, Inc.			Code ACI-001		Supplier Part No. A-9514					8 2/11/98		FME/		EA N NEA	io. -001							
Part Name Filter								Design Responsibil Brad Anderson	ity				Applica Sedan /	tion/Model Yea 1998	Model Year										
Core Team Brad Ander	1 Ison, Jerry Ben	ware,Lisa Br	own,Ken Caracci,Bill Co	x,Fre	ed Jo	ordan,Ken Kratz					Prepared E Brad A. And	sy Jerson				Date 2/1	9 1/98								
tiem /	Po	itential	Potential	8	C I	Potential Cause(s) /	0	Current	•	R	Recommended	Responsibili	ity	Action Re		scutts									
Function		Failure Effect(c) of Mode Failure		Ŷ	8 6	Mechanisms of Failure	u r	Design Controls	e o	P. N.	Action(s)	& Target Completion D	late	Actions Taken	e v	•	ŧ	P.							
Filter for assembly w	Iter for Insufficient wax ssembly with coverage over		Deteriorated life of door leading to:	4	ਂ	Insufficient wax thickness specified	4	Supplier certification	1	16	None	N/A 2/11/98													
644 to frewall	vali specified	a specified surface o	appearance due rust through pai	a n	appearance due to rust through paint	appearance due to rust through paint	appearance due to rust through paint	appearance due to rust through paint	appearance due to rust through paint	appearance due to rust through paint			Inappropriate wax specified	5	set up set up	4	80								
			function of Interior door hardware					Five piece setup, in-process, end of run study	2	40	None	N/A 2/11/98													
	Corroded lower doo	Interior r panels	Improper oxide coating	6	Ð	Entrapped air prevents wax from entering corner/edge access	6	Test spray pattern at startup and after idle periods, and	5	180	Add team evaluation using production spray equipment and specified wax	Engineering and Assembly Operations 2/18/98	1 Ba reu #9: mo	sed on test its (Test 989) spray head diffed to	6	2	5	60							
					U	Spray heads clogged: Viscosity too high,	4	Incomming audit per 200-16 certification, SPC Lot/Qtr	2	48															
								Temperature too low, Pressure too low		Laboratory test using "worst case" wax and application	3	72	Add laboratory accelerated corrosion testing	ABC Labs 2/27/98	Tel spi	st results show echied	6	3	3	54					
							hole size			Conduct DOE on wax thickness	Engineering Associates 2/18/98	DC var spe this acc	E shows 25% tation in ectiled skness is xeptable	6	2	2	24								
						Feeder not	3											Γ							
			L			property or		ļ																	

	Approved By Brad A. Anderson	Date 2/11/98
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FMEA CRITERIAS

CRITERIAS FOR PROBABILITY OF OCCURANCE (Po)	Frequency	Rating
It is most improbable that the defect appears. E.g. fool-proof construction	< 1 / 100 000	1
Very low probability that the defect appears, similar construction without defect	< 1 / 10 000	2 - 3
Low probability that the defect appears	< 1 / 1 000	4 - 5
Certain probability that the defect appears	< 1 / 100	6 - 7
High probability that the defect appears	< 1 / 10	8 - 9
Very high probability that the defect appears	< 1 / 1	10

CRITERIAS FOR SEVERITY (S)	Rating
No accident hazard or effect to the product	1
No accident hazard or insignificant effect to the product, still intact function	2 - 3
Very low accident hazard or risk of interference of the function in the production	4 - 6
Accident hazard under special circumstances or problem with the function in the production	7 - 8
Serious risk for personal injury	9 - 10

CRITERIAS FOR PROBABILITY OF DETECTION (Pd)	Rating
Defect which will be always noticed	1
Normal probability for detection (at c:a 75% intensity of defect)	2 - 3
Certain probablity for detection (at c:a 50% intensity of defect)	4 - 6
Low probability for detection - alt. No part of the control programme	7 - 8
It is improbable that the defect will be detected - can not be tested	9 - 10



Discussion and conclusions of FMEA method

- reveals most weaknesses of the system
 - no guarantee to find all critical failures due to:
 - lack of imagination & ability to identify possible problems
 - human errors often overlooked
- systematic overview of the failures
- basis for quantitative analyses (fault tree)





- unsuitable for analysing systems with much redundancy
- all components are analysed (including failures of little or no consequence)
- Sub-systems could be defined
- Computer tools for execution of FMEA



Storage tank



Top event "Over-filling of the tank"



Top event"Undesiredstoppage of the fluid supply"





Minimal cut sets

A cut set of a fault tree is a set of basic events the occurrence of witch ensures that the top event occurs

Cut sets for the tree:

 $\mathbf{5}$

 $\mathbf{2}$

3

 $\{1,5\}\ \{4,5\}\ \{1,2,3\}\ \{2,3,4\}$





Quantitative analysis of the fault tree

Probabilities of basic events taken from the tables



Cut sets for the tree:

 $\{1,5\}$ $\{4,5\}$ $\{1,2,3\}$ $\{2,3,4\}$

0.02x0.01+0.01x0.01+0.02x0.02x0.02+0.01x0.02x0.02 =

= 0.03 x 10⁻² = 0.03%



Alternative tank analysis diagram







