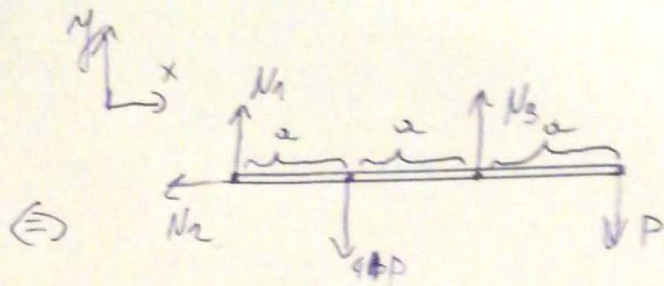
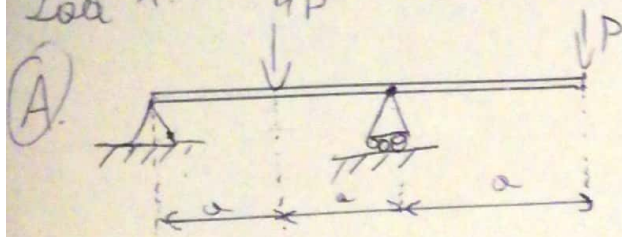


Zadatak 1.



$P = \frac{10+8}{50} \cdot 10 = 0,36 \text{ kN}$   
 $a = 5 + \frac{8}{50} \cdot 10 = 5,16 \text{ m}$   
 $P = 10 + \frac{8}{50} \cdot 10 = 10,16 \text{ kN}$

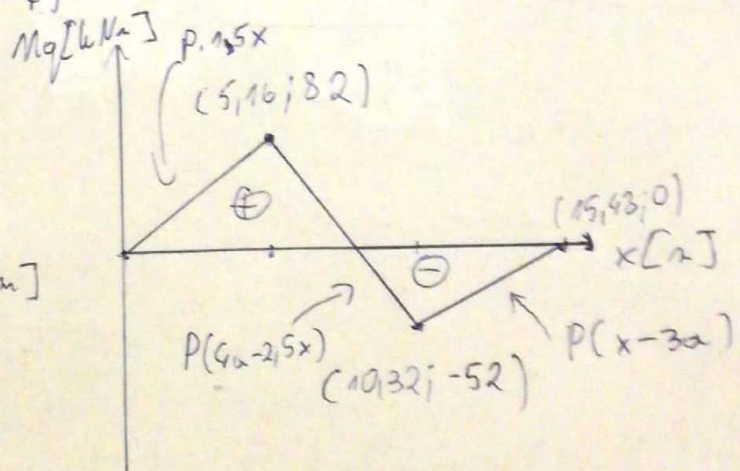
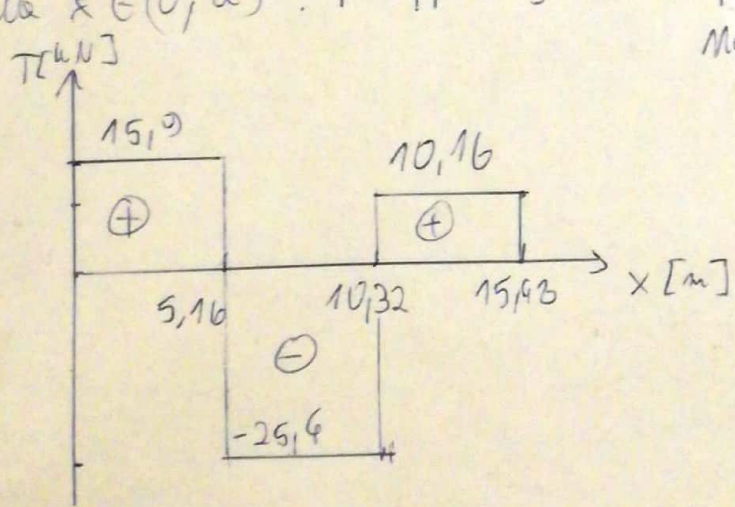
na osi x  
 ravnoteza sila  
 ravnoteza momenta

$$\begin{cases} N_2 = 0 \\ N_1 + N_3 = 5P \\ -4P \cdot a + N_3 \cdot 2a + P \cdot 3a = 0 \end{cases}$$

$\hookrightarrow N_3 = \frac{7}{2} P = 3,5P$   
 $N_1 = 1,5P$

Razljud sily tnapaj:

dla  $x \in (2a, 3a)$ :  $T - P = 0 \Rightarrow T = P = 10,16 \text{ kN}$   
 dla  $x \in (a, 2a)$ :  $T + N_3 - P = T + 2,5P = 0 \Rightarrow T = -2,5P = -25,4 \text{ kN}$   
 dla  $x \in (0, a)$ :  $T - 4P + N_3 - P = T - 1,5P = 0 \Rightarrow T = 1,5P = 15,9 \text{ kN}$

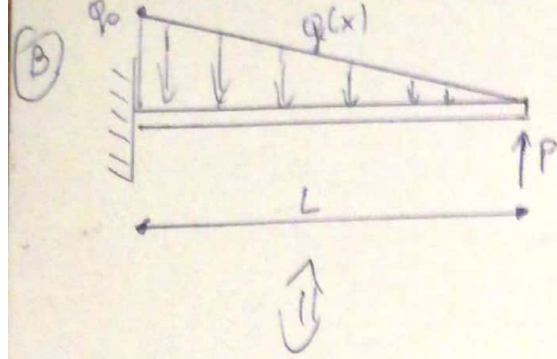


Razljud momenta naprepa:

dla  $x \in (2a, 3a)$   
 $-M_q - P(3a-x) = 0$   
 $M_q = P(x-3a) = 10,16(x-15,48) \text{ [kNm]}$

dla  $x \in (0, a)$   
 $M_q - N_1 x = 0$   
 $M_q = 1,5P x = 15,9 x \text{ [kNm]}$

dla  $x \in (a, 2a)$   
 $M_q + 4P(x-a) - N_1(x-a) = 0$   
 $M_q + P(4x-4a-1,5x) = 0$   
 $M_q = P(4a-2,5x)$   
 $M_q = 10,16(20,64 - 2,5x)$

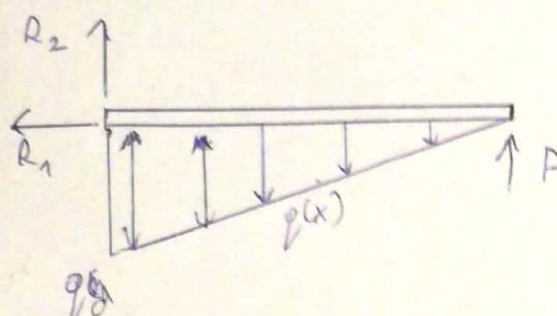


$$q_0 = 5 + \frac{5}{50} = 5,16 \text{ kN/m}$$

$$P = 84 + \frac{5}{50} = 4,1 \text{ kN}$$

$$L = 2 \text{ m}$$

~~q(x) = \frac{q\_0}{L} x~~

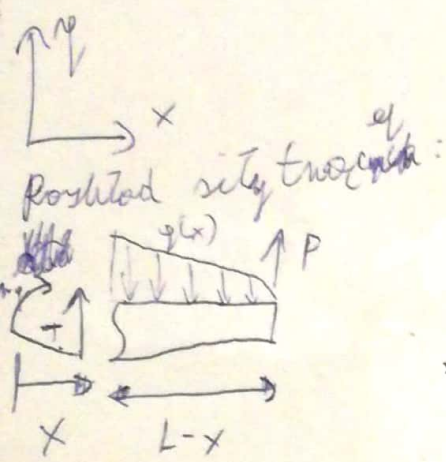


$$\sum F_{xi} = 0 \Rightarrow R_1 = 0$$

$$\sum F_{yi} = 0 \Rightarrow R_2 + P - \frac{1}{2} q_0 L = 0$$

$$R_2 = \frac{1}{2} q_0 L - P$$

$$R_2 = \frac{1}{2} \cdot 2 \cdot 5,16 - 4,1 = 1,06 \text{ kN}$$



~ podobnostna trojkektar

$$T + P - \frac{1}{2} q_0 L \cdot \left(\frac{L-x}{L}\right)^2 = 0$$

$$T = \frac{1}{2} q_0 \frac{(L-x)^2}{L} - P$$

$$T = \frac{1}{4} 5,16 (2-x)^2 - 4,1 = (1,29(2-x)^2 - 4,1) \text{ [kN]}$$

Rozklad momentu trojcektar:

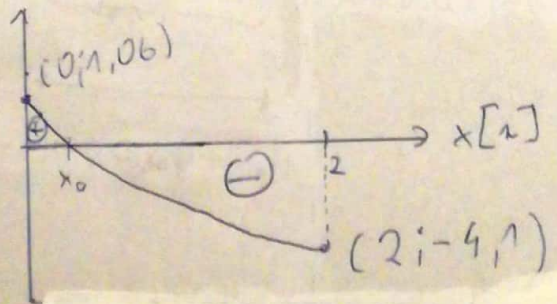
$$-M_q + P(L-x) + \frac{1}{3} (L-x) \cdot \frac{1}{2} q_0 L \left(\frac{L-x}{L}\right)^2 = 0$$

sr. vishosini
najs. Δ
stara podobnostna Δ

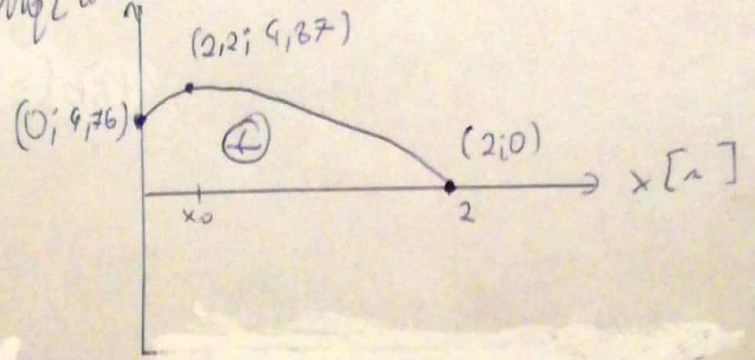
$$M_q = (L-x) \left( P + \frac{1}{6} q_0 \frac{(L-x)^2}{L} \right)$$

$$M_q = (2-x) \left( 4,1 + \frac{1}{12} 5,16 (2-x)^2 \right) = (2-x) (4,1 + 0,43(2-x)^2) \text{ [kNm]}$$

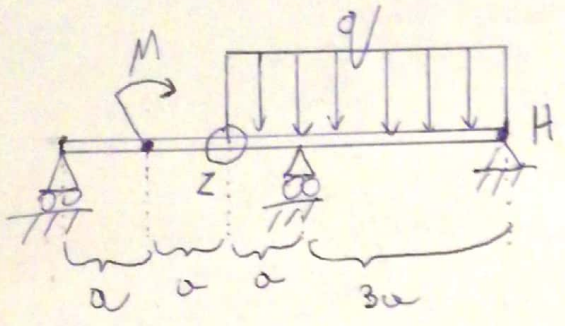
T [kN]



M<sub>q</sub> [kNm]



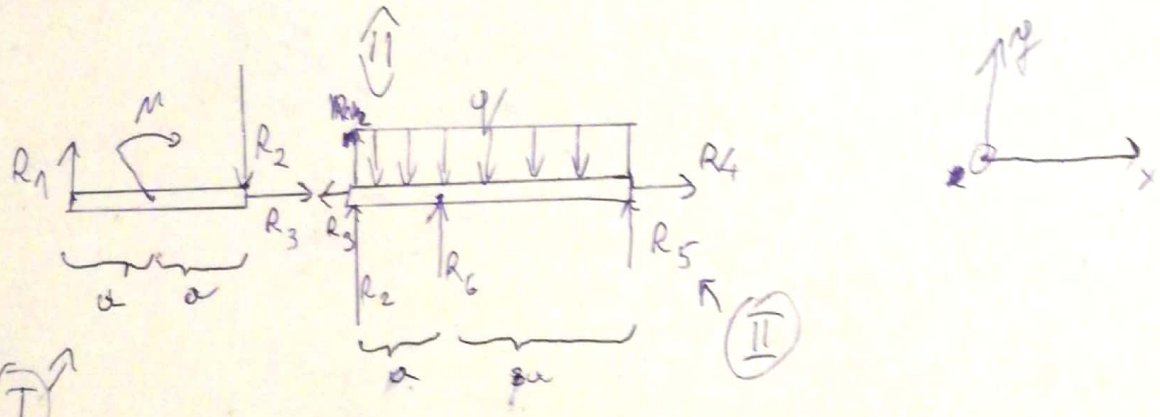
C



$$q = 10 + \frac{5}{50} = 10,1 \text{ kN/m}$$

$$M = 2 + \frac{8}{50} = 2,16 \text{ kNm}$$

$$a = 0,5 \text{ m}$$



(I)

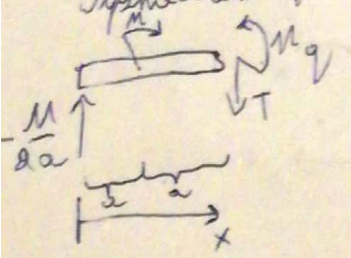
Wyznaczenie reakcji:

$$\sum F_x = 0 \Rightarrow R_3 = 0$$

$$\sum F_y = 0 \Rightarrow R_1 - R_2 = 0 \Rightarrow R_1 = R_2$$

$$\sum M_z = 0 \Rightarrow -M - 2a R_2 = 0 \Rightarrow R_2 = -\frac{M}{2a}$$

(wzrostowa lewostronna)  
Wyznaczenie siły q i momentu qrcy:



$$\sum F_x = 0 \Rightarrow -T + \left(-\frac{M}{2a}\right) = 0$$

$$T = -\frac{M}{2a}$$

$$T = -2,16 \text{ kN}$$

$$\sum M_z = 0:$$

dla  $x \in (0, a)$ :

$$-\left(-\frac{M}{2a}\right)x + M_q = 0$$

$$M_q = -\frac{M}{2a} \cdot x = -2,16x \text{ [kNm]}$$

dla  $x \in (a, 2a)$ :

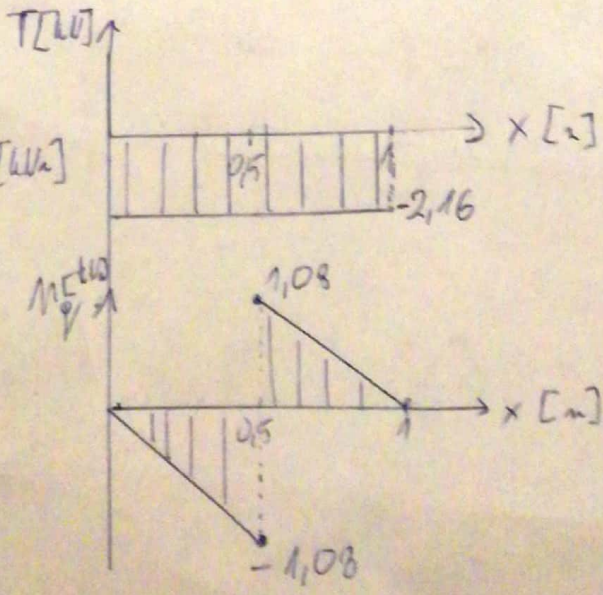
$$M_q - \left(-\frac{M}{2a}\right)x - M = 0$$

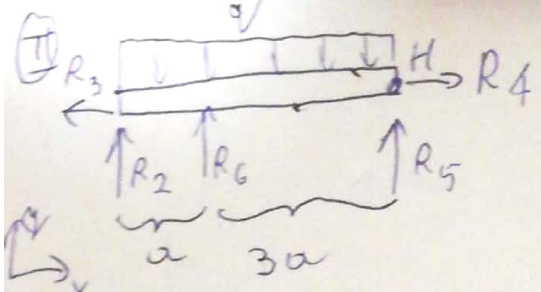
$$M_q = M - \frac{M}{2a}x = 2,16\left(1 - \frac{x}{2a}\right) \text{ [kNm]}$$

$$M_q(a) = -\frac{M}{2}$$

$$M_q(a) = M - \frac{M}{2} = \frac{M}{2}$$

$$M_q(2a) = M - M = 0$$





2 oblišené (I)

$$R_2 = -\frac{M}{2a} = -\frac{2,16 \text{ kNm}}{1 \text{ m}} = -2,16 \text{ kN}$$

$$R_3 = 0$$

Významom reakcie:

$$\sum F_x = 0 \Rightarrow R_4 + R_3 = 0 \quad R_4 = 0$$

$$\sum M_H = 0 \Rightarrow R_6 \cdot 3a + R_2 \cdot 4a - q \cdot 4a \cdot 2a = 0$$

$$R_6 \cdot 3a = 8a^2 \cdot q - 4a \cdot R_2$$

$$R_6 = \frac{1}{3} (8 \cdot 10,1 \frac{\text{kN}}{\text{m}} + 2,16 \text{ kN})$$

$$R_6 = \frac{1}{3} (80,8 \text{ kN} + 8,64 \text{ kN}) \approx 16,35 \text{ kN}$$

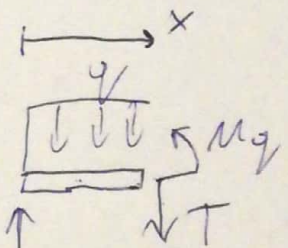
Významom T a Mg:

ako  $x \in (2a, 3a)$ :

$$R_2 - T - q(x - 2a) = 0$$

$$T = -q(x - 2a) + R_2 = 7,94 - 10,1x$$

$$T = -10,1(x - 1) + 2,16 = 10,1x - 10,1 + 2,16 = 10,1x - 7,94$$



$$M_g - R_2(x - 2a) + q(x - 2a) \frac{(x - 2a)}{2} = 0$$

$$M_g = R_2(x - 2a) - \frac{q}{2}(x - 2a)^2 = 0$$

$$M_g = -2,16(x - 1) + \frac{10,1}{2}(x - 1)^2 = 5,05x^2 - 7,21x + 7,31$$

$$M_g = -5,13x^2 + 8,99x - 3,14$$

ako  $x \in (3a, 6a)$ :

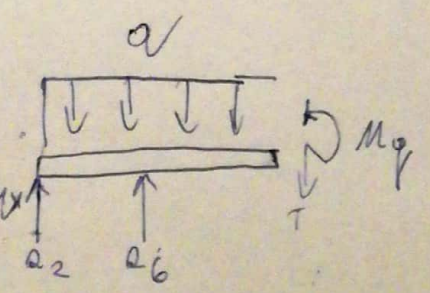
$$T: R_2 + R_6 - T - q(x - 2a) = 0$$

$$T = -q(x - 2a) + R_2 + R_6$$

$$T = -10,1(x - 1) + 2,16 + 16,35 = 18,81 - 10,1x$$

$$T = 22,13 - 10,1x$$

$$M_g = M_g - R_2(x - 2a) - R_6 \cdot 3$$



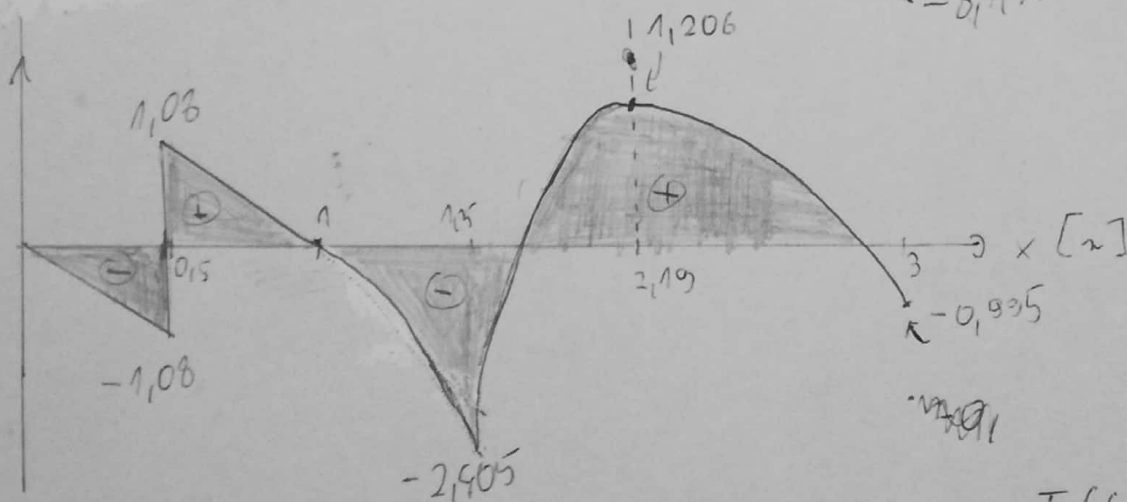
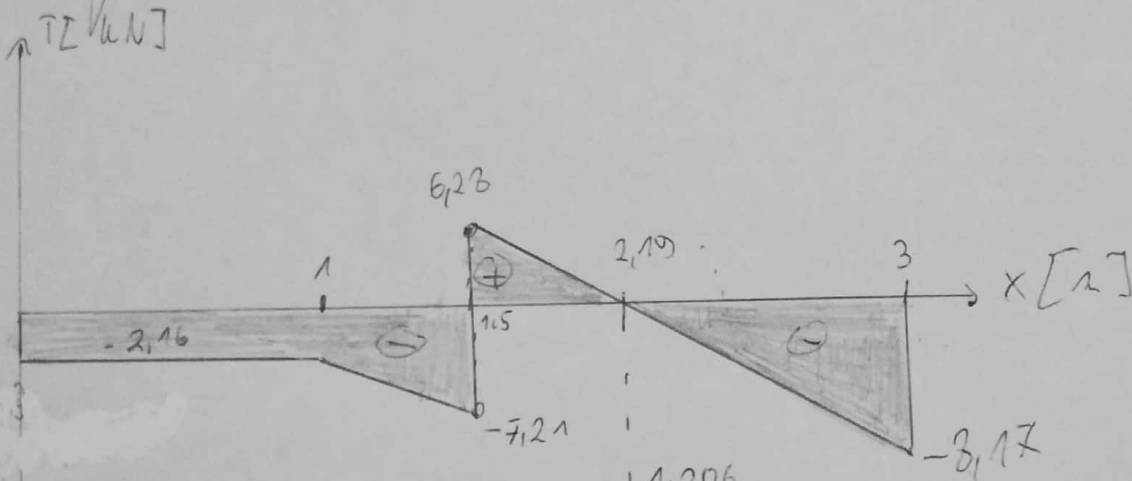
$$M_{\varphi}:$$

$$M_{\varphi} + q(x-2a) \frac{(x-2a)}{2} - R_6(x-3a) - R_2(x-2a) = 0$$

$$M_{\varphi} = R_2(x-2a) + R_6(x-3a) - q \cdot \frac{1}{2}(x-2a)^2$$

$$M_{\varphi} = -2,16(x-1) + 6,28(x-1,5) - \frac{10,1}{2}(x-1)^2$$

$$M_{\varphi} = -5,3x^2 + 29,79x - 27,665$$



$$T(2a) = -2,16 \text{ kN}$$

$$T(2a) = -2,16 \text{ kN}$$

$$M(2a) = -1,08 \text{ kNm}$$

$$M(2a) = 1,08 \text{ kNm}$$

$$M(2a) = 0 \text{ kNm}$$

$$M(2a) = 0 \text{ kNm}$$

$$T(3a) = -7,21 \text{ kN}$$

$$T(3a) = 6,28 \text{ kN}$$

$$M(3a) = -2,405 \text{ kNm}$$

$$M(3a) = -2,405 \text{ kNm}$$

$$M(4,38a) = 1,206 \text{ kNm}$$

$$M(6a) = -0,995 \text{ kNm}$$

$$T(6a) = -8,17$$

$$T(x) = 0 \Rightarrow 10,1x = 22,13$$

$$x = 2,19 \approx 4,38a$$

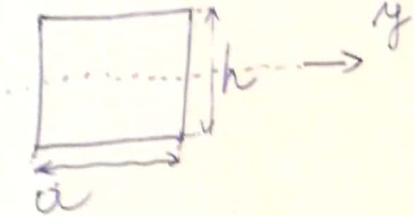
Zad 2.

$k_r = 200 \text{ MPa}$

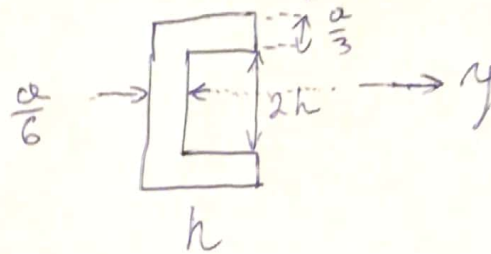
Z zad 1A:  $M_{max} = 15,9 \text{ kNm}$

$a = 1,2h$

Przekrój I



Przekrój II



$$y_I = \frac{1}{12} a h^3 = \frac{1,2}{12} h^4 = 0,1 h^4$$

$$y_{II} = \frac{1}{12} h \cdot (2h + \frac{2}{3}a)^3 - \frac{1}{12} (h - \frac{a}{6}) (2h)^3 = \frac{1}{12} (h \cdot (2,8h)^3 - 0,8h \cdot 3h^3)$$

$$= 1,296 h^4$$

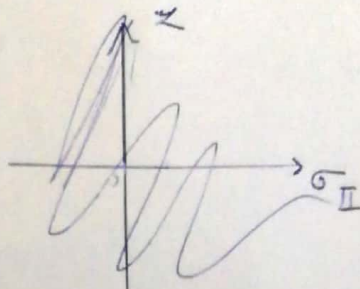
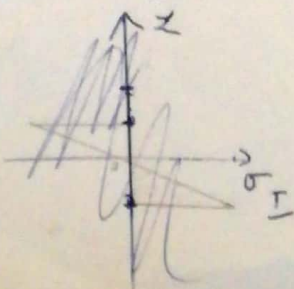
$k_r \leq \sigma$       $k_r = |\sigma_{extr}|$       $\sigma_{extr} = \frac{-M_{max} z_{max}}{y}$

$$I: k_r = \left| \frac{-M_{max} \cdot \frac{h}{2}}{y_I} \right| = \frac{15,9 \cdot \frac{h}{2}}{0,1 h^4} \Rightarrow h = \sqrt[3]{\frac{79 \text{ kNm}}{200 \text{ MPa}}} \approx 7,35 \text{ cm}$$

$$II: k_r = \frac{15,9 \cdot 1,4 h}{1,296 h^4} \Rightarrow h = \sqrt[3]{\frac{15,9 \cdot 1,4}{200}} \approx 4,94 \text{ cm}$$

$y_{y_{II}} = 3782 \text{ cm}^4$

$$\sigma_{extr II} = \frac{-15,9 \text{ kNm} \cdot 1,4 h}{3782 \text{ cm}^4} = \frac{-15,9 \text{ kNm} \cdot 1,4}{5191,6 \text{ cm}^3} \approx 43,26 \text{ MPa} < \sigma_{extr I}$$



~~$\sigma_{I,II}$~~

$$L_{\max I} = \frac{h}{2} = 3,675 \text{ m}$$

$$L_{\max II} = 1,9h = 10,29 \text{ m}$$

$$\sigma_{\max I} = 200 \text{ MPa}$$

$$\sigma_{\max II} = 93,26 \text{ MPa}$$

