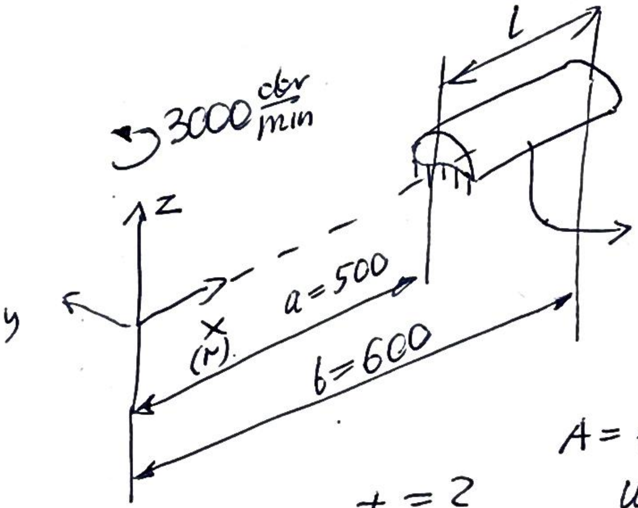
Łopatka turbiny

Po jakim czasie łopatka wydłuży się o $u_{dop} = 2mm$?
Przedstawić na wykresie przemieszczenia wybranych punktów łopatki w funkcji czasu.




$E = 1.7 \cdot 10^5\ MPa$

$\alpha = 1{,}29 \cdot 10^{-5}\ \frac{1}{K}$

$\rho = 7.6 \cdot 10^{-9}\ \frac{Ns^2}{mm^4}$

$B = 3 \cdot 10^{-11}\ \frac{1}{MPa^n \cdot h}$

$n = 2.6$

$\Delta T = 600°C$

$A = 100\ mm^2$

$u(b, t_k) = 2mm$

$\omega = \frac{\pi n}{30}$

$t_k = ?$

Wyniki symulacji porównać z teoretycznymi:

$$\Delta l(\omega) = \int_a^b \varepsilon(r)\,dr = \int_a^b \frac{\rho\omega^2}{2E}(b^2-r^2)\,dr = \frac{\rho\omega^2}{6E}(2b^3+a^3-3b^2a)$$

$$\Delta l(\Delta T) = \alpha \Delta T \cdot L$$

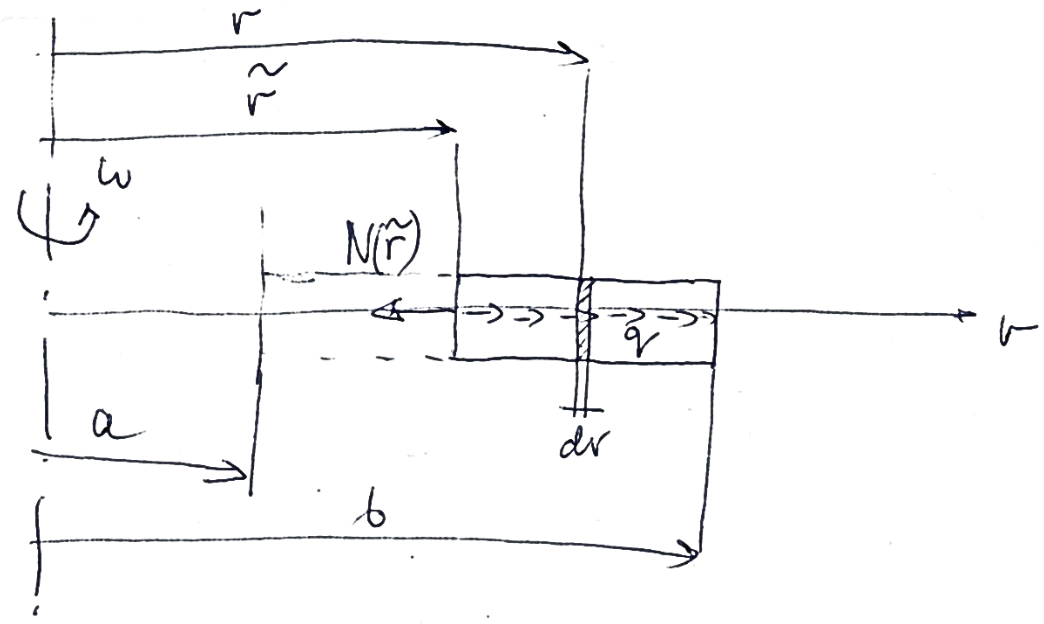
$$\Delta l_c(t) = \int_a^b \varepsilon_c\,dr = \int_a^b B\sigma^n t\,dr = \int_a^b B\left[\frac{\rho\omega^2}{2}(b^2-r^2)\right]^n \cdot t\,dr =$$

$$= Bt\left(\frac{\rho\omega^2}{2}\right)^n \int_a^b (b^2-r^2)^n\,dr = K \cdot t$$

$$\Delta l(\omega) + \Delta l(\Delta T) + \Delta l_c(t) \leq u_{dop}$$

$$\Delta l(\omega) + \Delta l(\Delta T) + K \cdot t_k \leq u_{dop}$$

$$t_k \leq \frac{u_{dop} - \Delta l(\omega) - \Delta l(\Delta T)}{K}$$





$$dB = dm\cdot\omega^2 r = $$
$$= \rho\, dr\cdot A\omega^2 r$$

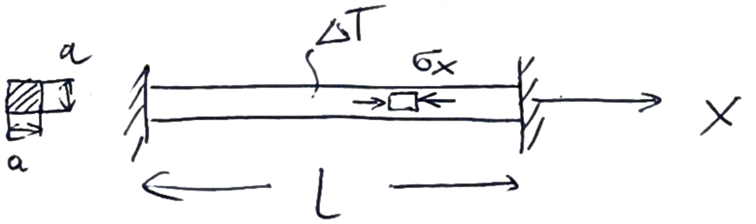
Wydatek sił masowych : $\quad q = \frac{dB}{dr} = \rho A\omega^2 r$

$$\sum F_r = 0 \Rightarrow N(\tilde{r}) = \int_{\tilde{r}}^{b} q\, dr = \rho A\omega^2 \frac{r^2}{2}\Big|_{\tilde{r}}^{b} =$$
$$= \frac{1}{2}\rho A\omega^2 (b^2 - \tilde{r}^2) \Rightarrow N(r) = \frac{1}{2}\rho A\omega^2 (b^2 - r^2)$$

$$u(\tilde{r}) = \int_a^{\tilde{r}} \varepsilon(r)\, dr = \int_a^{\tilde{r}} \frac{\sigma(r)}{E}\, dr = \int_a^{\tilde{r}} \frac{N(r)}{EA}\, dr =$$
$$= \frac{\rho\omega^2}{2E}\int_a^{\tilde{r}} (b^2 - r^2)\, dr = \frac{\rho\omega^2}{2E}\left(b^2 r\Big|_a^{\tilde{r}} - \frac{r^3}{3}\Big|_a^{\tilde{r}}\right) =$$
$$= \frac{\rho\omega^2}{2E}\left(b^2(\tilde{r} - a) - \frac{(\tilde{r}^3 - a^3)}{3}\right)$$

$$\Delta l(\omega) = u(b) = \frac{\rho\omega^2}{2E}\left(b^2(b-a) - \frac{b^3}{3} + \frac{a^3}{3}\right) = \frac{\rho\omega^2}{6}\left(2b^3 + a^3 - 3b^2 a\right)$$

Pręt ogrzany równomiernie. Dane materiałowe jak w poprzednim zadaniu z łopatką.




$$\sigma_y = \sigma_z = \tau_{ij} = 0$$

na kierunku X

$$\Delta L = \Delta L_e(t) + \Delta l_T + \Delta l_C(t) = 0 \quad \Big| : L$$

$$\frac{\Delta L}{L} = \varepsilon_e(t) + \varepsilon_T + \varepsilon_c(t) = 0$$

$$\varepsilon_e(t) = \frac{\sigma_x(t)}{E}, \quad \varepsilon_T = \alpha \Delta T, \quad \varepsilon_c(0) = 0$$

$\Rightarrow$ dla $t = 0$ $\quad \frac{\sigma_x(0)}{E} + \alpha\Delta T = 0 \Rightarrow \sigma_x(0) = -E\alpha\Delta T$

ściskanie
będzie relaksacja

Prawo Nortona

$$\frac{d\varepsilon_c}{dt} = B \cdot \sigma_{eq}^n$$

naprężenie zredukowane

bo tu nie może być ujemnej wartości w argumencie funkcji wykładniczej.

np. hipoteza Hubera:

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2\right] + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$$= \sqrt{\sigma_x^2} = |\sigma_x| = \begin{cases} \sigma_x & \text{dla } \sigma_x \geq 0 \\ -\sigma_x & \text{dla } \sigma_x < 0 \end{cases}$$

$$\sigma_{eq}(t) = -\sigma_x(t)$$

$$\Rightarrow \sigma_x(t) = -\sigma_{eq}(t)$$

$$\frac{\sigma_x(t)}{E} + \alpha \cdot \Delta T + \varepsilon_c(t) = 0$$

$$-\frac{\sigma_{eq}(t)}{E} + \alpha\Delta T + \varepsilon_c(t) = 0 \quad \Big| \cdot \frac{d}{dt}$$

$$-\frac{1}{E}\frac{d\sigma_{eq}}{dt} + 0 + \frac{d\varepsilon_c}{dt} = 0$$

$$-\frac{1}{E}\frac{d\sigma_{eq}}{dt} + B \cdot \sigma_{eq}^n = 0 \quad \Big| \cdot E$$

$$BE\sigma_{eq}^n = \frac{d\sigma_{eq}}{dt}$$

$$dt = \frac{1}{BE}\frac{d\sigma_{eq}}{\sigma_{eq}^n} \quad \Big| \int$$

$$\int_0^{\tilde{t}} dt = \frac{1}{BE} \int_{\sigma_{eq}(0)}^{\sigma_{eq}(\tilde{t})} \frac{d\sigma_{eq}}{\sigma_{eq}^{n}} \Rightarrow$$

$$\tilde{t} = \frac{1}{BE(n-1)} \left( \frac{1}{(\sigma_{eq}(\tilde{t}))^{n-1}} - \frac{1}{(\sigma_{eq}(0))^{n-1}} \right)$$

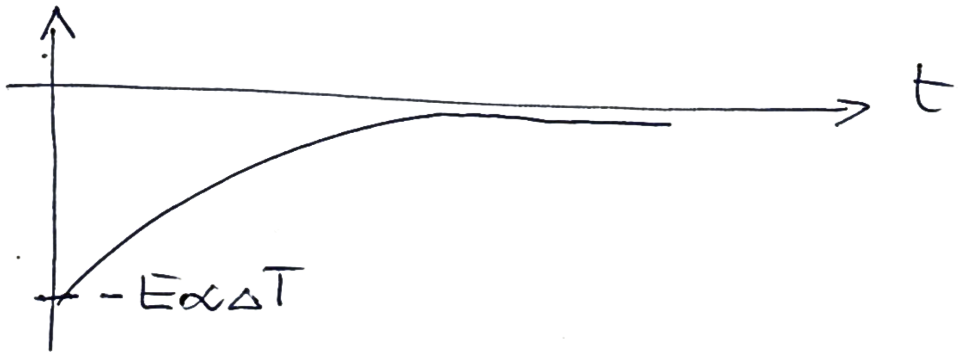
$$BE(n-1) \cdot \tilde{t} + \frac{1}{(E\alpha\Delta T)^{n-1}} = \frac{1}{(\sigma_{eq}(\tilde{t}))^{n-1}}$$

do potęgi

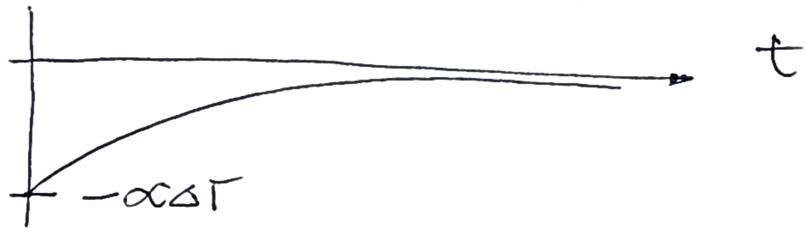
$$(\sigma_{eq}(\tilde{t}))^{n-1} = \frac{1}{BE(n-1)\tilde{t} + \frac{1}{(E\alpha\Delta T)^{n-1}}} \Bigg|^{\frac{1}{n-1}} \ldots$$

$$\sigma_{eq}(t) = \left( \frac{1}{BE(n-1) \cdot t + \frac{1}{(E\alpha\Delta T)^{n-1}}} \right)^{\frac{1}{n-1}}$$

$$\sigma_x(t) = -\sigma_{eq}(t)$$

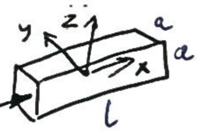



$$\varepsilon_x(t) = \varepsilon_e(t) = -\frac{\sigma_{eq}(t)}{E}$$



ODKSZTAŁCENIA:

$\underline{t=0}$




$\sigma_x = -\alpha \Delta T E$
$\sigma_y = 0$
$\sigma_z = 0$

odkształcenia termiczne:

$\varepsilon_{xT} = \varepsilon_{yT} = \varepsilon_{zT} = \alpha \cdot \Delta T$

odkształcenia sprężyste:

$\varepsilon_{xe} = -\varepsilon_{xT} = -\alpha \Delta T$

$\varepsilon_{ye} = -\nu \varepsilon_{xe} = \nu \alpha \Delta T$

$\varepsilon_{ze} = \varepsilon_{ye} = \nu \alpha \Delta T$

odkształcenia pełzania:

$\varepsilon_{xc} = 0, \quad \varepsilon_{yc} = 0, \quad \varepsilon_{zc} = 0$

odkształcenia całkowite:

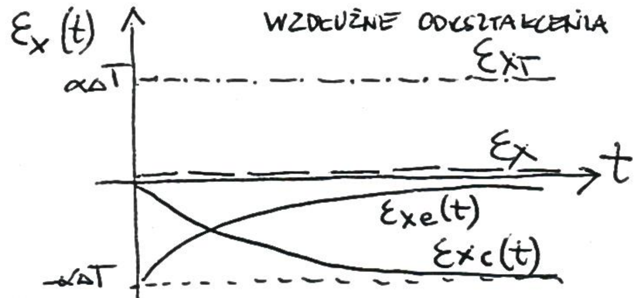
$\varepsilon_x = \varepsilon_{xT} + \varepsilon_{xe} + \varepsilon_{xc} = 0$

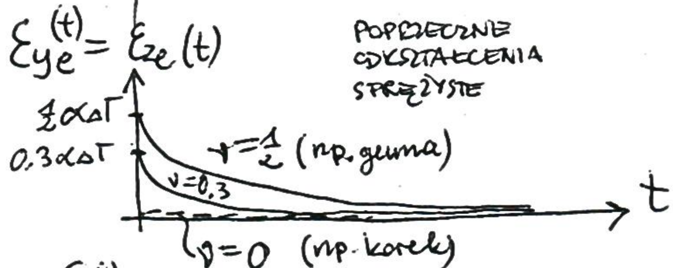
$\varepsilon_y = \varepsilon_{ye} + \varepsilon_{yT} + \varepsilon_{yc} = (1+\nu)\alpha \Delta T$

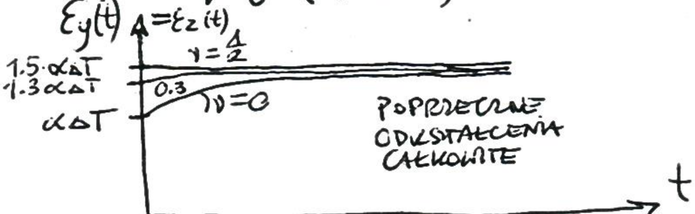
$\varepsilon_z = \varepsilon_y = (1+\nu)\alpha \Delta T$

względna zmiana objętości:

$\Delta V / V = \varepsilon_x + \varepsilon_y + \varepsilon_z = 2(1+\nu)\alpha \Delta T$

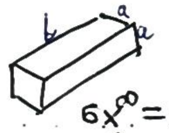




$t = 2400h$ (zakładamy, że czas jest na tyle długi, że jest to rozwiązanie bliskie uzyskanemu dla $t \to \infty$)




$\left.\begin{array}{l} \sigma_x^\infty = 0 \\ \sigma_y^\infty = 0 \\ \sigma_z^\infty = 0 \end{array}\right\}$ relaksacja całkowita

odkształcenia termiczne:

$\varepsilon_{xT} = \varepsilon_{yT} = \varepsilon_{zT} = \alpha \Delta T$

odkształcenia sprężyste:

$\varepsilon_{xe}^\infty = \varepsilon_{ye}^\infty = \varepsilon_{ze}^\infty = 0$

odkształcenia pełzania:

$\varepsilon_{xc}^\infty$ – można policzyć, wiedząc, że odkształcenia całkowite $\varepsilon_x^\infty = 0$

$\varepsilon_{xc}^\infty = \varepsilon_x^\infty - \varepsilon_{xT} - \varepsilon_{xe}^\infty = -\alpha \Delta T$

$\varepsilon_{yc}^\infty$ i $\varepsilon_{zc}^\infty$ – można policzyć, zakładając, że nastąpiła swobodna dylatacja termiczna (stan bez naprężeń):

$$\left.\frac{\Delta V}{V}\right|_\infty = \overset{0}{\varepsilon_x^\infty} + \varepsilon_y^\infty + \varepsilon_z^\infty = 3 \cdot \alpha \Delta T$$

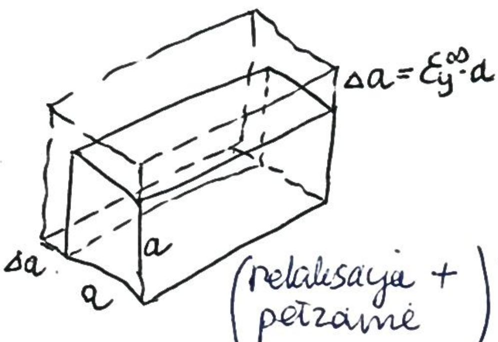
$\varepsilon_y^\infty = \varepsilon_{ye}^\infty + \varepsilon_{yT} + \varepsilon_{yc}^\infty = 0 + \alpha \Delta T + \varepsilon_{yc}^\infty$

$\varepsilon_z^\infty = \varepsilon_{ze}^\infty + \varepsilon_{zT} + \varepsilon_{zc}^\infty = 0 + \alpha \Delta T + \varepsilon_{zc}^\infty$

$\varepsilon_{yc}^\infty = \varepsilon_{zc}^\infty$ stąd:

$2\alpha \Delta T + \varepsilon_{yc}^\infty + \varepsilon_{zc}^\infty = 3\alpha \Delta T$

$\varepsilon_{yc}^\infty = \varepsilon_{zc}^\infty = \frac{1}{2} \cdot \alpha \Delta T$




(relaksacja + pełzanie)