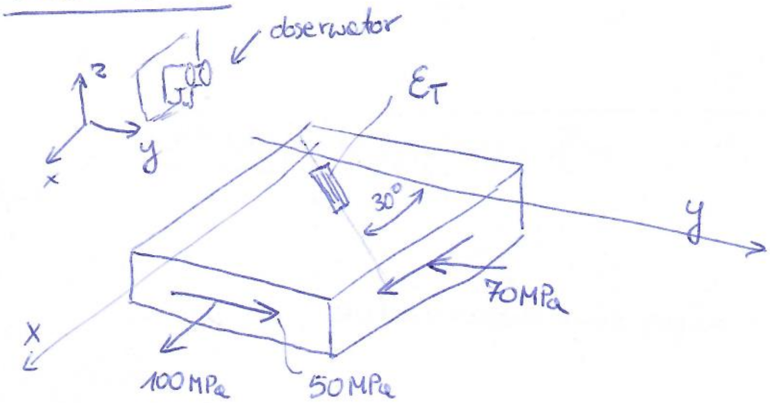
Zadanie 3



$E = 0{,}7 \cdot 10^5$ MPa

$\nu = 0{,}3$

$R_e = 280$ MPa

$n_e = 1{,}5$

Znaleźć:

1.) $\sigma_1, \sigma_2, \sigma_3$ (przedstawić graficznie)

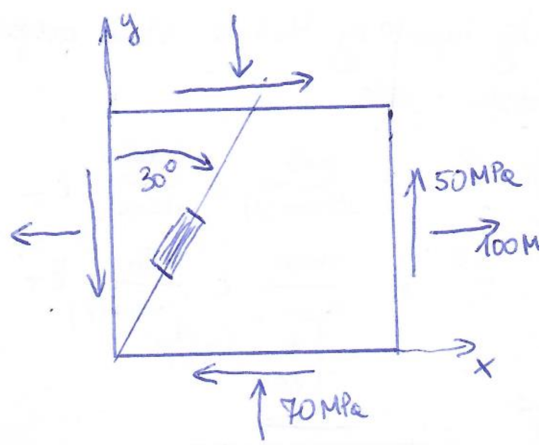
2.) σ_{red}^H, σ_{red}^T

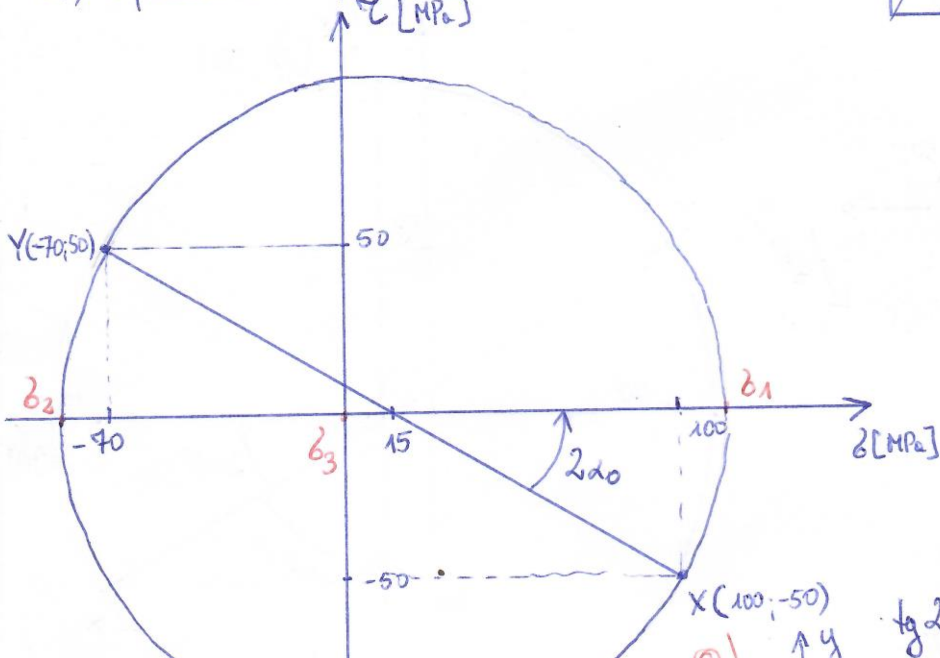
3.) ε_T

4.) $\frac{\Delta V}{V}$

5.) Sprawdzić bezpieczeństwo

Widok z punktu widzenia obserwato





$\sigma_m = 15$ MPa

$R = \sqrt{\left(\frac{\cancel{100+70}}{2}\right)^2 + 50^2} \simeq 99$M

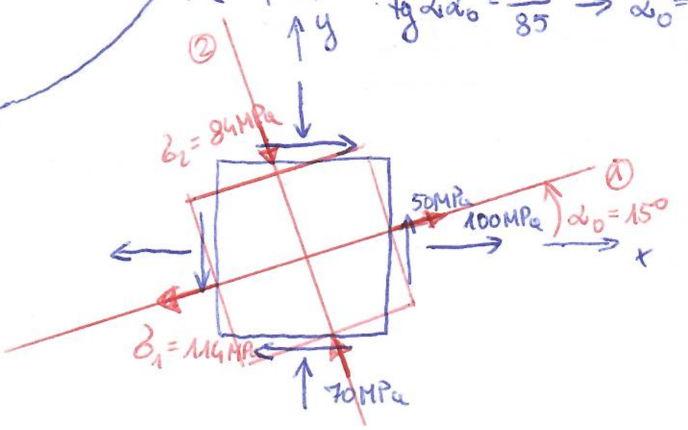
$(100-15)^2$

1.) $\sigma_1 = \sigma_m + R = 15 + 99 =$

$\sigma_2 = \sigma_m - R = 15 - 99 = -8$

$\sigma_3 = 0$

$\mathrm{tg}\, 2\alpha_0 = \frac{50}{85} \rightarrow \alpha_0 \simeq 15^0$



2.) $\tau_{max} = R = 99$ MPa występuje w płaszczyźnie 1-2

$$\sigma_{red}^{T} = 2\,\tau_{max} = 2\cdot 99 = \boxed{198\ \text{MPa}}$$

$$\sigma_{red}^{H} = \sqrt{\frac{1}{2}\left[(100+70)^2+(70-0)^2+(0-100)^2\right]+3\left(50^2+0_q+0\right)} \simeq \boxed{171\ \text{MPa}}$$

5.)

$$k_r = \frac{Re}{ne} = \frac{280}{1{,}5} \simeq \boxed{187\ \text{MPa}}$$ - naprężenia dopuszczalne

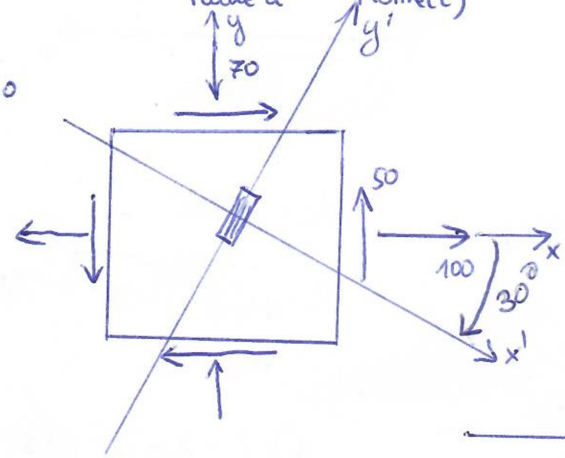
$$\sigma_{red}^{H} < k_r \quad , \quad \sigma_{red}^{T} > k_r$$

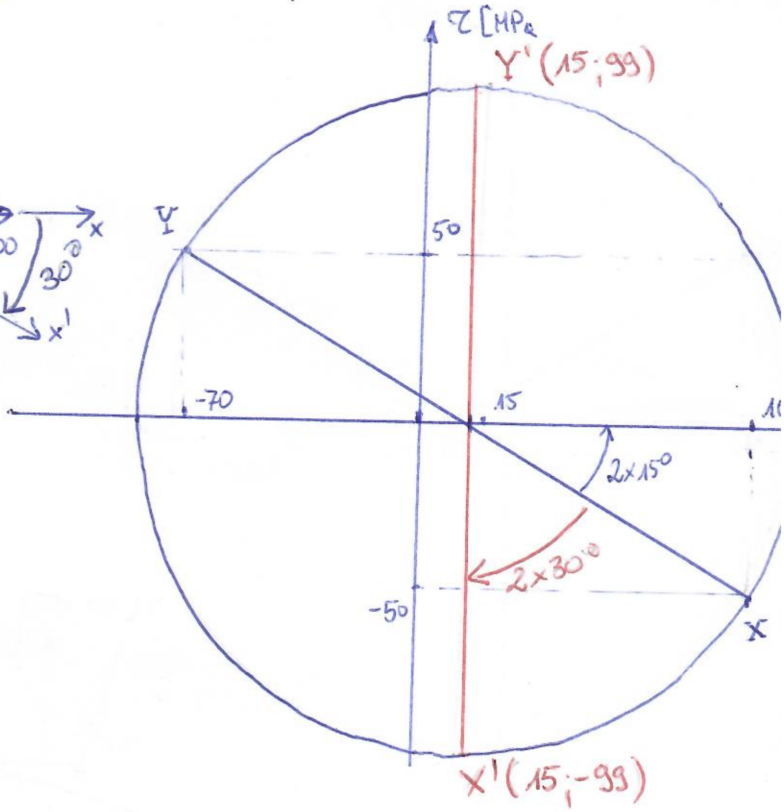
Wg hipotezy Hubera stan naprężenia jest bezpieczny, a wg hipotezy Treski - nie.

3.) 1° $\sigma \xrightarrow[\text{Mohra}(\sigma)]{\text{koło}} \sigma' \xrightarrow[\text{Hooke'a}]{\text{prawo}} \varepsilon_T$

2° $\sigma \xrightarrow[\text{Hooke'a}]{\text{prawo}} \varepsilon \xrightarrow[\text{Mohra}(\varepsilon)]{\text{koło}} \varepsilon_T$

1°

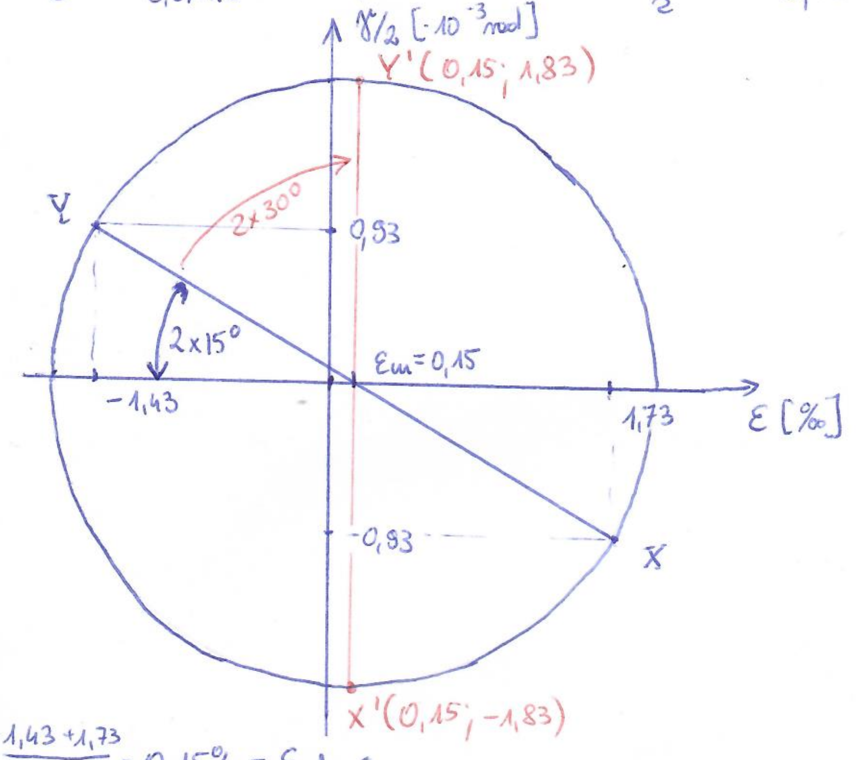




$$\varepsilon_T = \varepsilon_{y'} = \frac{1}{E}\left(\sigma_{y'}' - \nu\sigma_{x'}'\right) =$$

$$= \frac{1}{0{,}7\cdot 10^5}\left(15 - 0{,}3\cdot 15\right) = \underline{0{,}15‰}$$

2°
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu \sigma_y)$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu \sigma_x)$$
$$\varepsilon_x = \frac{1}{0{,}7\cdot10^5}(100 + 0{,}3\cdot70) = 1{,}73\ ‰$$
$$\varepsilon_y = \frac{1}{0{,}7\cdot10^5}(-70 - 0{,}3\cdot100) = -1{,}43\ ‰$$
$$G = \frac{E}{2(1+\nu)} = \frac{0{,}7\cdot10^5}{2\cdot1{,}3} \simeq 0{,}27\cdot10^5\ \text{MPa}$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-50}{0{,}27\cdot10^5} = -1{,}85\cdot10^{-3}\,\text{rad} \rightarrow \frac{\gamma_{xy}}{2} = -0{,}93\cdot10^{-3}\,\text{rad}$$



$$\varepsilon_m = \frac{-1{,}43+1{,}73}{2} = \underline{0{,}15‰} = \varepsilon_{y'} = \varepsilon_T$$
$$R = \sqrt{0{,}93^2 + 1{,}58^2} = \sqrt{0{,}86 + 2{,}5} \simeq 1{,}83\ ‰$$

5.)
$$\varepsilon_z = \frac{1}{E}\left(-\nu(\sigma_x + \sigma_y)\right)$$
$$\varepsilon_z = \frac{1}{0{,}7\cdot10^5}(-0{,}3\cdot30) = -0{,}13\ ‰$$

$$e = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = 1{,}73‰ - 1{,}43‰ - 0{,}13‰ = \underline{0{,}17‰}$$

Zadanie 3

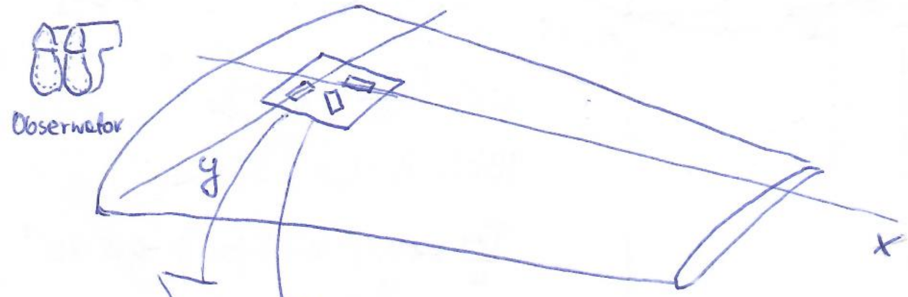
Na skrzydle samolotu naklejono rozetkę tensometrów i podczas lotu zarejestrowano wskazanie $\varepsilon_a, \varepsilon_b, \varepsilon_c$.

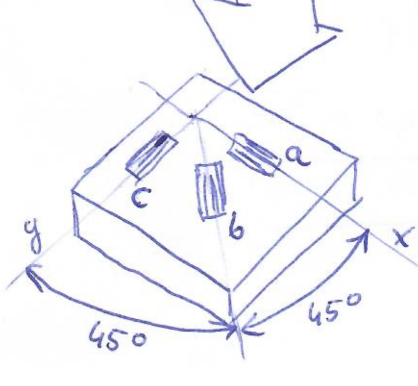
Durel: $E = 0{,}7 \cdot 10^5$ MPa, $\nu = 0{,}3$

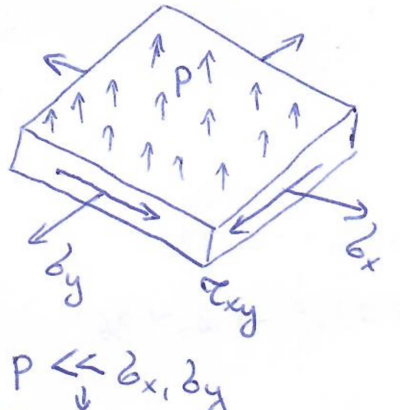
$\boxed{\begin{array}{l} \varepsilon_a = 0{,}4\text{‰} \\ \varepsilon_b = -0{,}2\text{‰} \\ \varepsilon_c = 0{,}1\text{‰} \end{array}}$

Wyznaczyć stan naprężenia:

1.) $\sigma_x, \sigma_y, \tau_{xy}$
2.) σ_1, σ_2
3.) τ_{max}

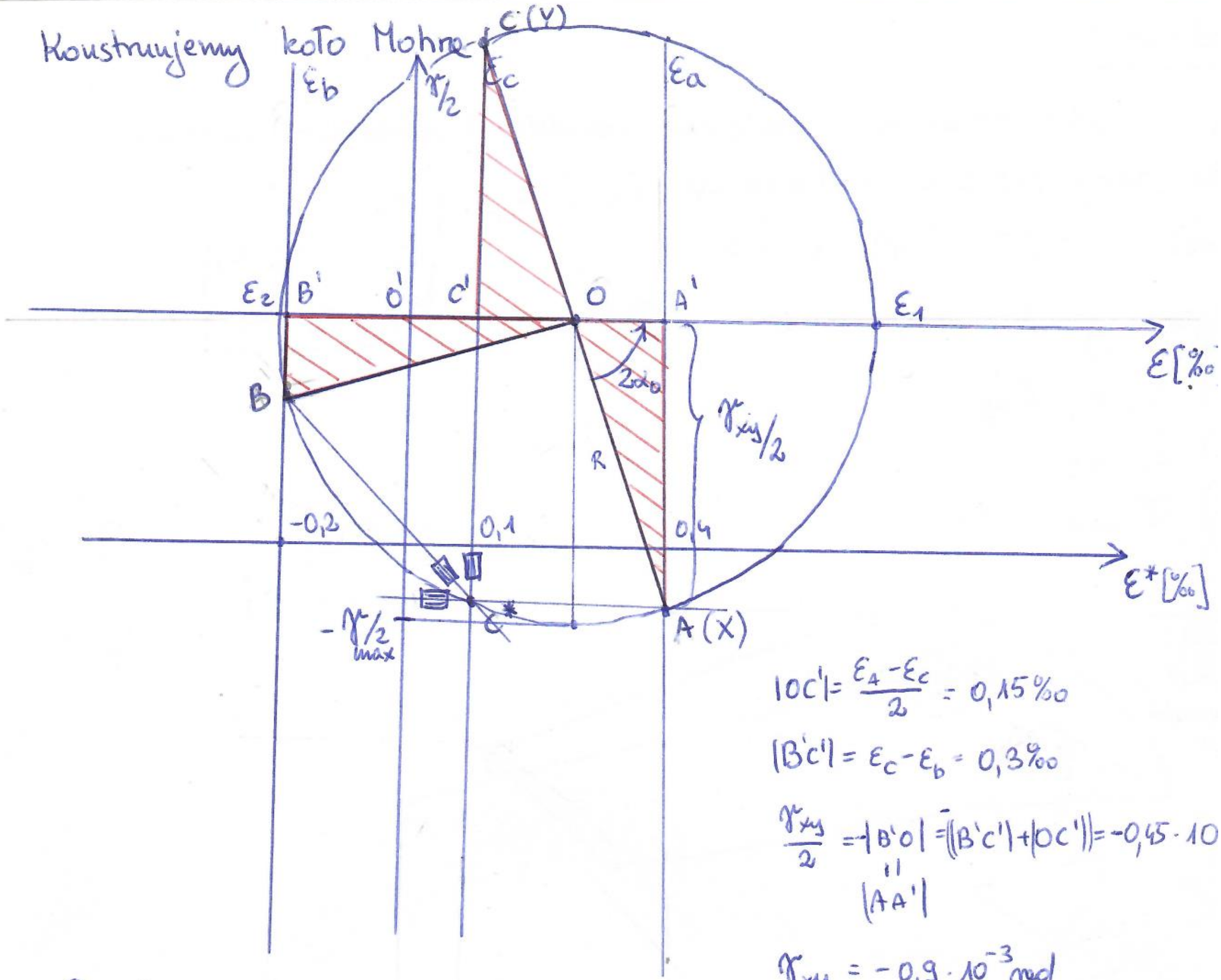






$p \ll \sigma_x, \sigma_y$
$\downarrow$
Płaski stan naprężenia
(PSN)

Konstruujemy koło Mohra



$|OC'| = \frac{\varepsilon_a - \varepsilon_c}{2} = 0{,}15‰$

$|B'C'| = \varepsilon_c - \varepsilon_b = 0{,}3‰$

$\frac{\gamma_{xy}}{2} = -\underset{\underset{|AA'|}{||}}{|B'O|} = -(|B'C'| + |OC'|) = -0{,}45 \cdot 10$

$\gamma_{xy} = -0{,}9 \cdot 10^{-3}\,\text{rad}$

$R = \sqrt{0{,}15‰^2 + 0{,}45‰^2} =$

$= \sqrt{0{,}0225 + 0{,}2025} \simeq 0{,}47‰$

$\varepsilon_x = \varepsilon_a = 0{,}4‰$

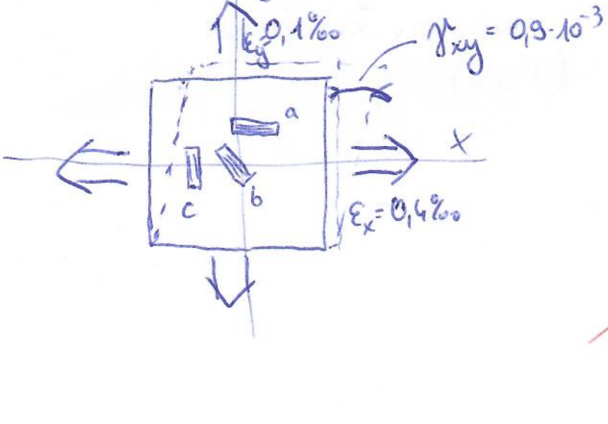
$\varepsilon_y = \varepsilon_c = 0{,}1‰$

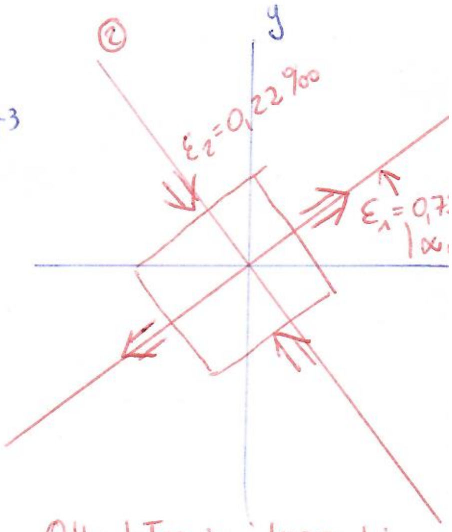
$\gamma_{xy} = -0{,}9 \cdot 10^{-3}\,\text{rad}$

$\varepsilon_1 = |OO'| + R = 0{,}25‰ + 0{,}47‰ = 0{,}72‰$

$\varepsilon_2 = |OO'| - R = 0{,}25\% - 0{,}47‰ = -0{,}22‰$

$\text{tg}\, 2\alpha_0 = \frac{0{,}45}{0{,}15} \rightarrow 2\alpha_0 \simeq 71{,}6° \rightarrow \alpha_0 \simeq 36°$





Odkształcenie i kierunki główne

Korzystamy z prawa Hooke'a dla PSN:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \qquad \xrightarrow[\text{względem } \sigma_x \text{ i } \sigma_y]{\text{Rozwiązanie}} \qquad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \qquad \qquad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

(warto zapamiętać)

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \qquad \tau_{xy} = G \cdot \gamma_{xy}$$

$$G = \frac{E}{2(1+\nu)} = \frac{0{,}7 \cdot 10^5}{2 \cdot 1{,}3} = 0{,}27 \cdot 10^5 \text{ MPa}$$

1.) $$\sigma_x = \frac{0{,}7 \cdot 10^5}{1-(0{,}3)^2}(0{,}4 \cdot 10^{-3} + 0{,}3 \cdot 0{,}1 \cdot 10^{-3}) \simeq 33 \text{ MPa}$$

$$\sigma_y = \frac{0{,}7 \cdot 10^5}{1-(0{,}3)^2}(0{,}1 \cdot 10^{-3} + 0{,}3 \cdot 0{,}4 \cdot 10^{-3}) \simeq 17 \text{ MPa}$$

$$\tau_{xy} = 0{,}27 \cdot 10^5 \cdot (-0{,}9 \cdot 10^{-3}) \simeq -24 \text{ MPa}$$

2.) $$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) = \frac{0{,}7 \cdot 10^5}{1-(0{,}3)^2}\left(0{,}72 \cdot 10^{-3} + 0{,}3 \cdot (-0{,}22 \cdot 10^{-3})\right) \simeq 50 \text{ MPa}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1) = \frac{0{,}7 \cdot 10^5}{1-(0{,}3)^2}(-0{,}22 \cdot 10^{-3} + 0{,}3 \cdot 0{,}72 \cdot 10^{-3}) \simeq -0{,}3 \text{ MPa}$$

3.) $$\tau_{max} = G \cdot \gamma_{max} = G \cdot R \cdot 2 = 0{,}27 \cdot 10^5 \cdot (0{,}47) 10^{-3} \cdot 2 \simeq -25 \text{ MPa}$$